Oracle Separation Between Quantum Commitments and One-Wayness

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joint with Barak Nehoran (Princeton University) and

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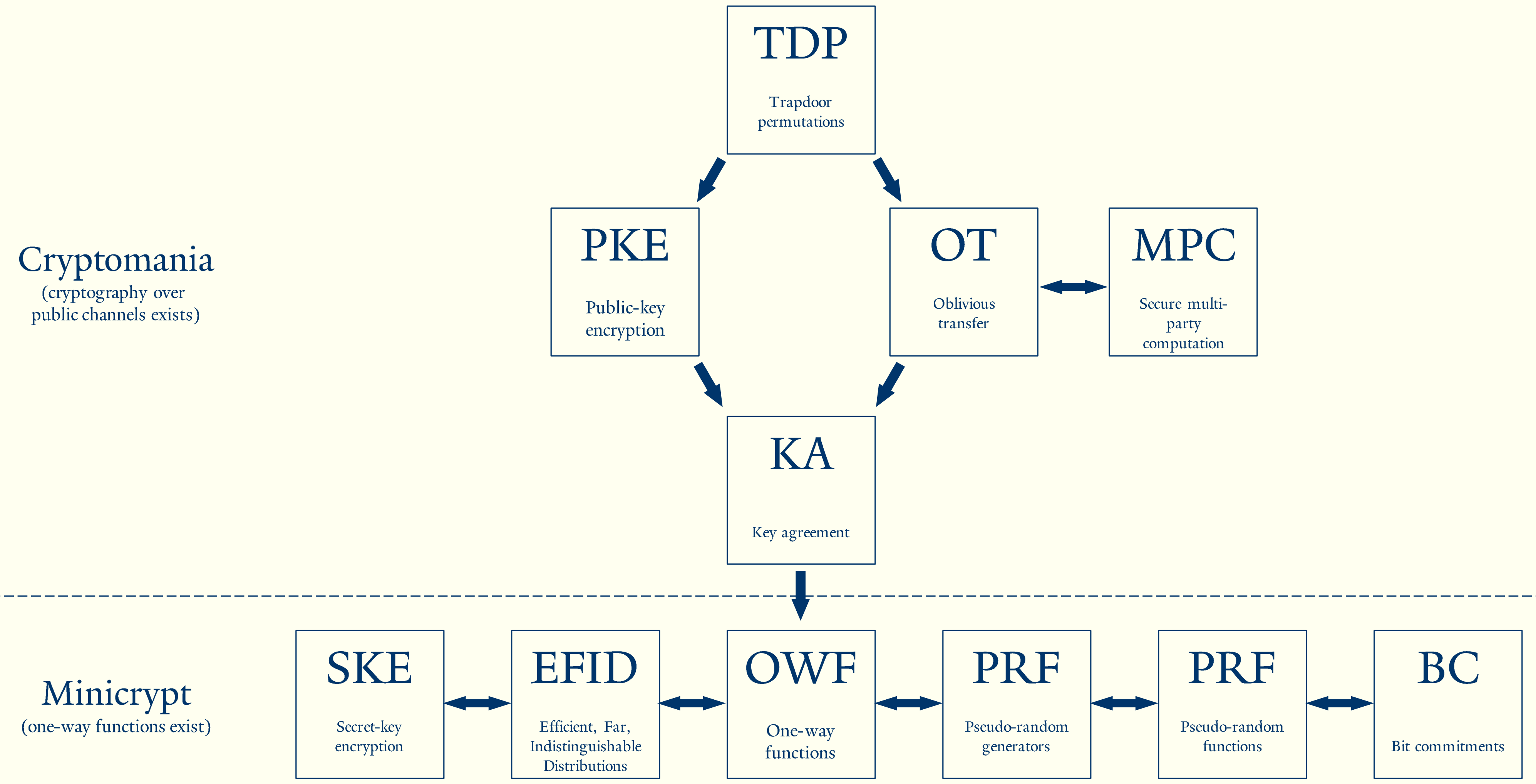
Landscape of (classical) cryptography



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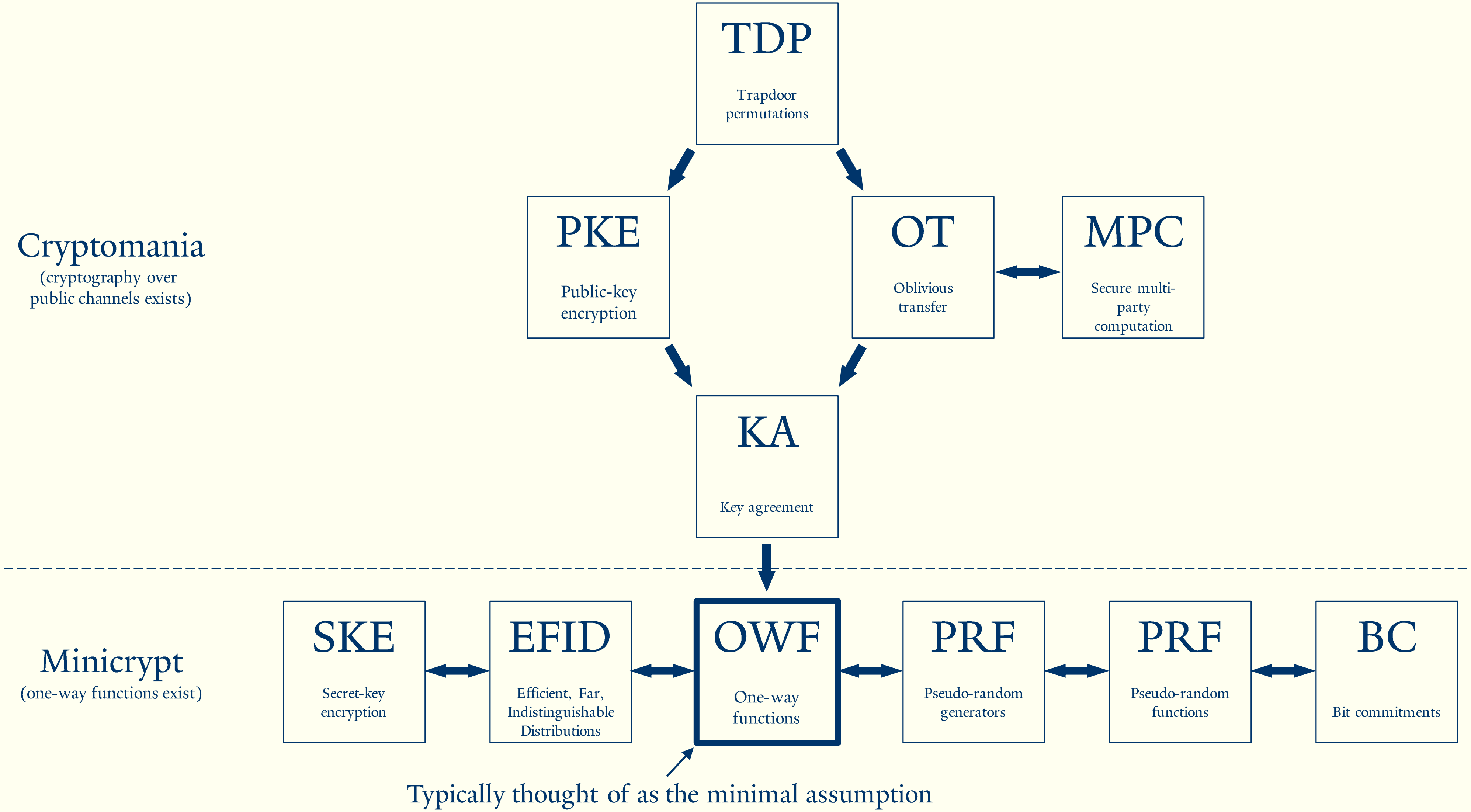
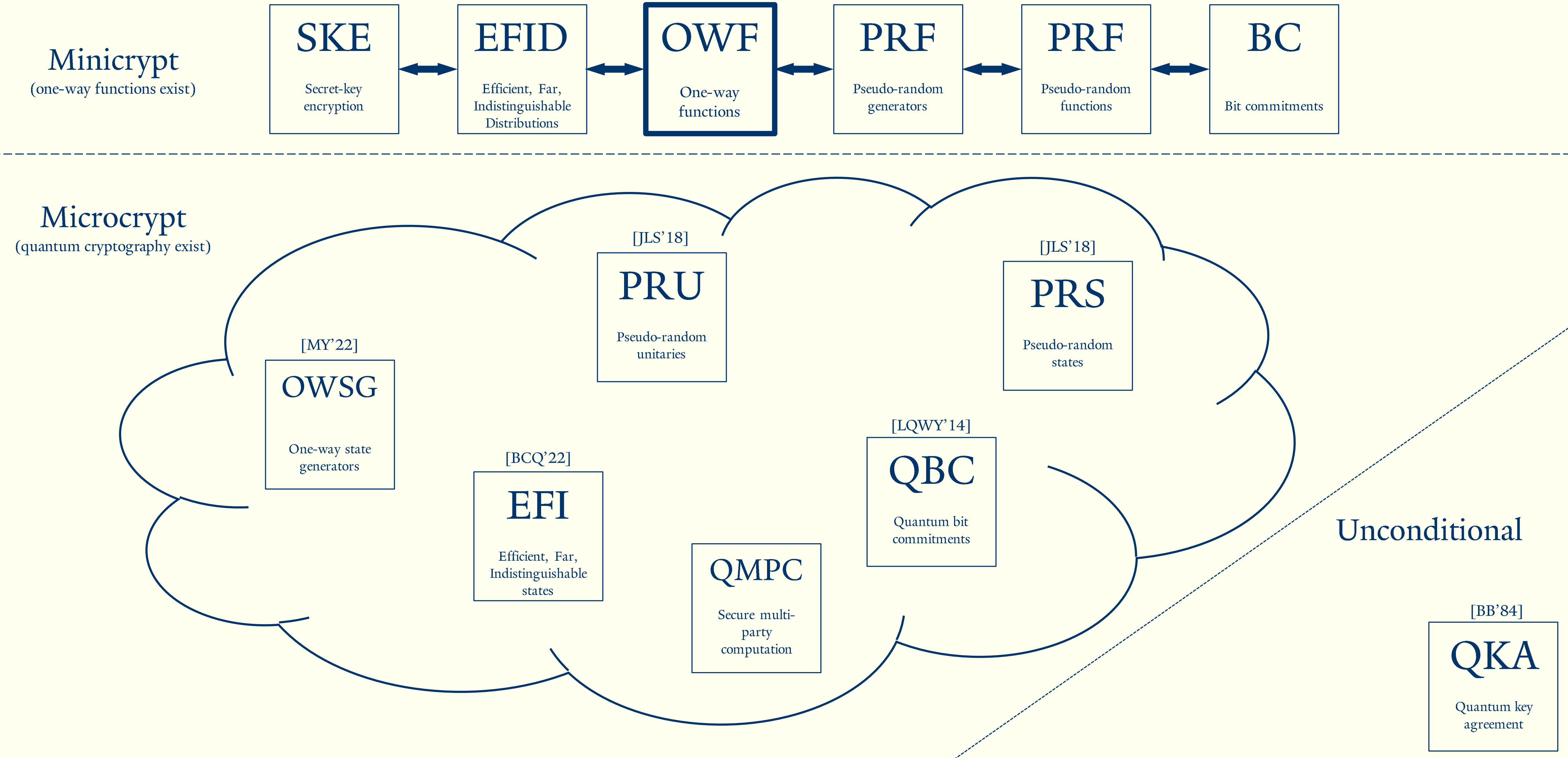
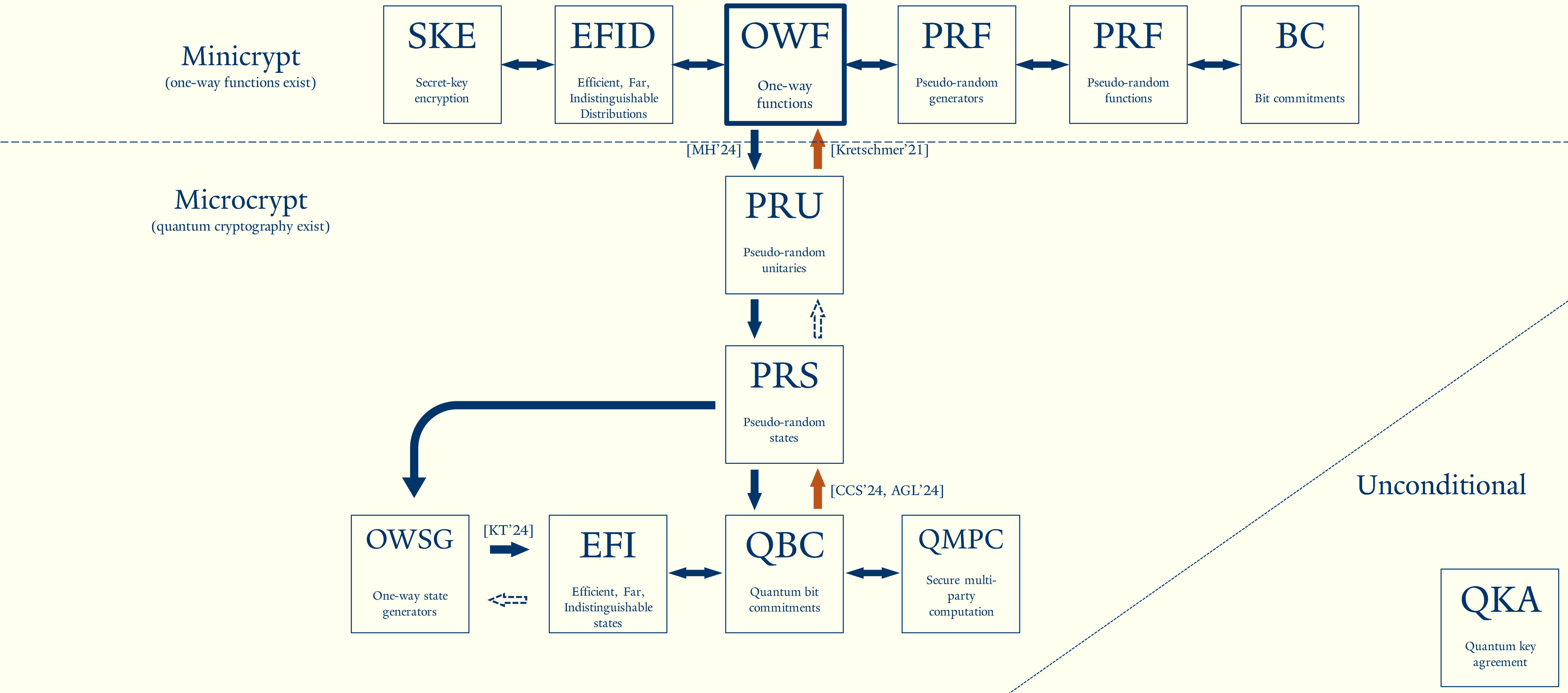


Landscape of quantum cryptography



Landscape of quantum cryptography



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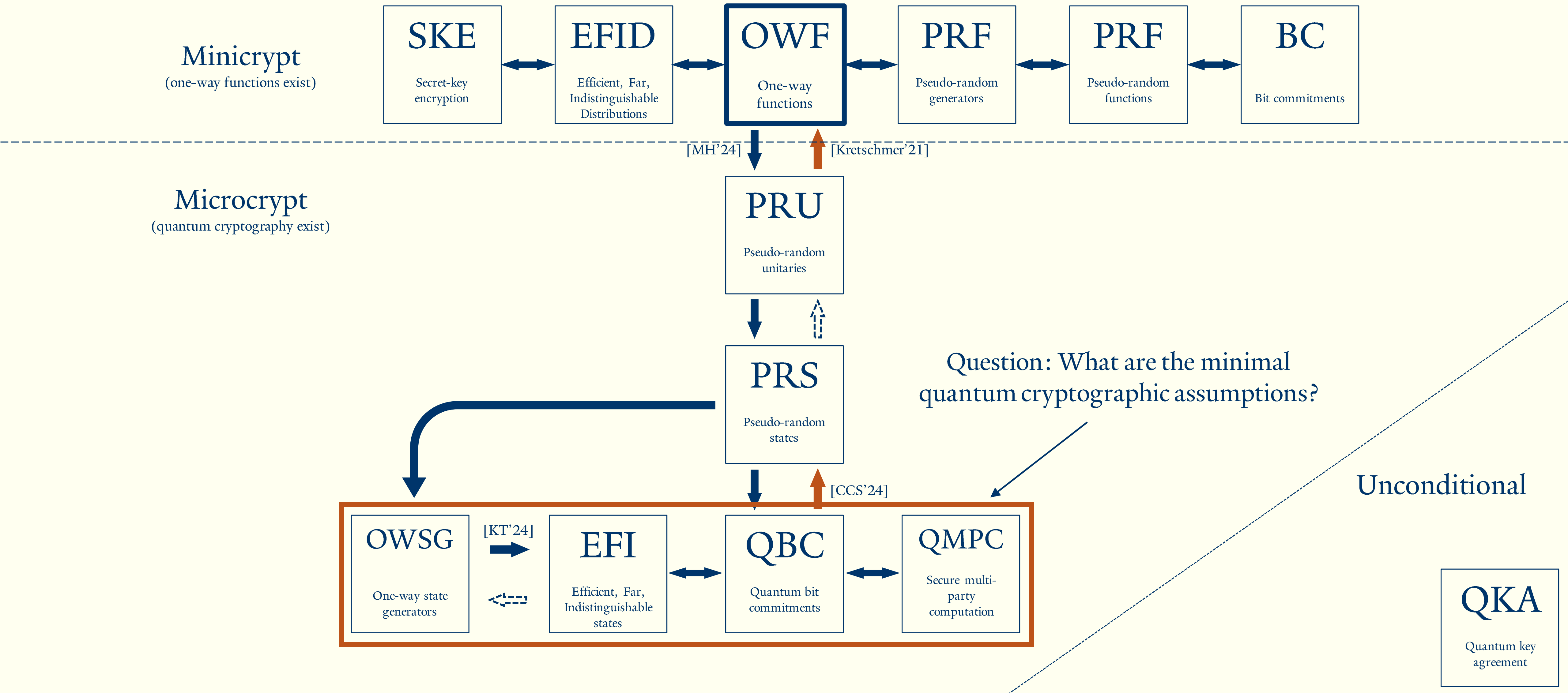

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One-way state generators

- (Correctness) There is an efficient algorithm $\mathrm{Ver}(k, \rho)$ such that

$$\Pr_k[\top \leftarrow \mathrm{Ver}(k, \rho_k)] \geq 1 - \mathrm{negl}(\lambda).$$

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$$\Pr_{k}[\top \leftarrow \mathrm{Ver}(k, \rho_k)] \geq 1 - \mathrm{negl}(\lambda).$$

- (Security) For all efficient adversaries A,

$$\Pr_{k}\left[\top \leftarrow \mathrm{Ver}(k', \rho_k) \mid k' \leftarrow \mathrm{A}(\rho_k^{\otimes t(\lambda)})\right] \leq \mathrm{negl}(\lambda).$$

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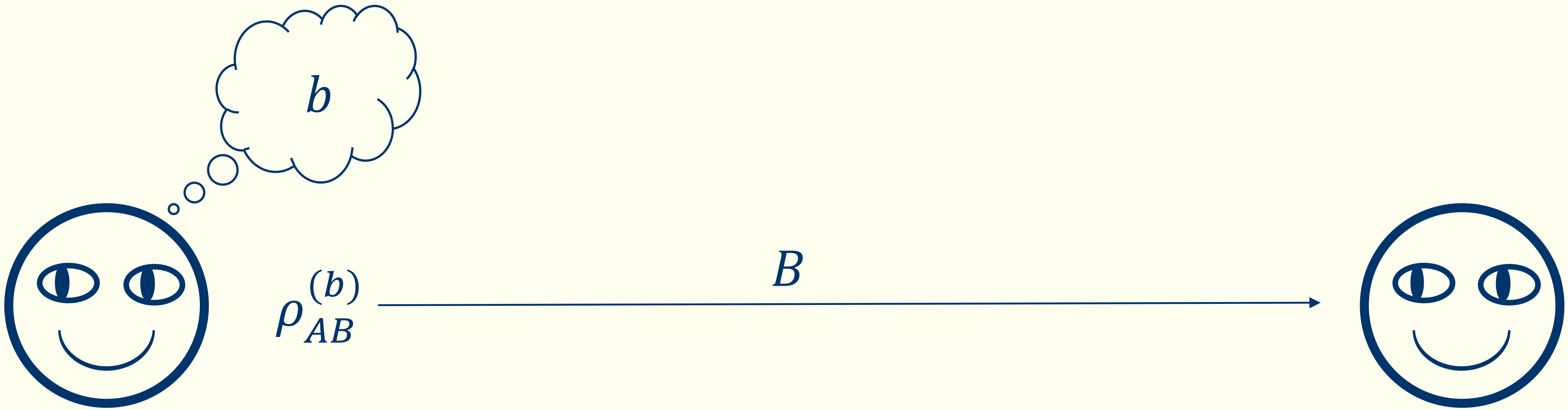

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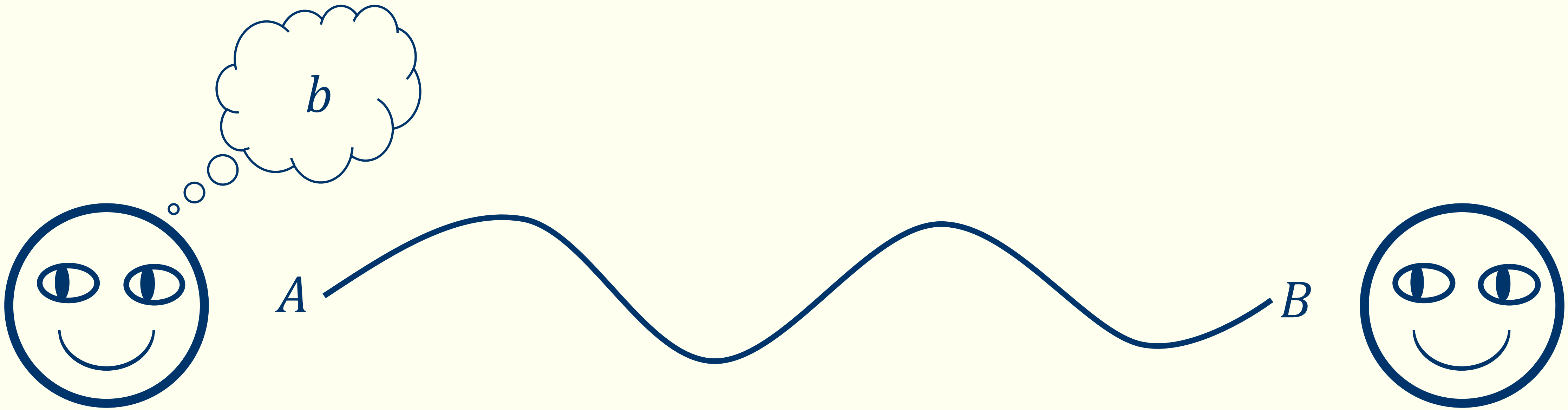

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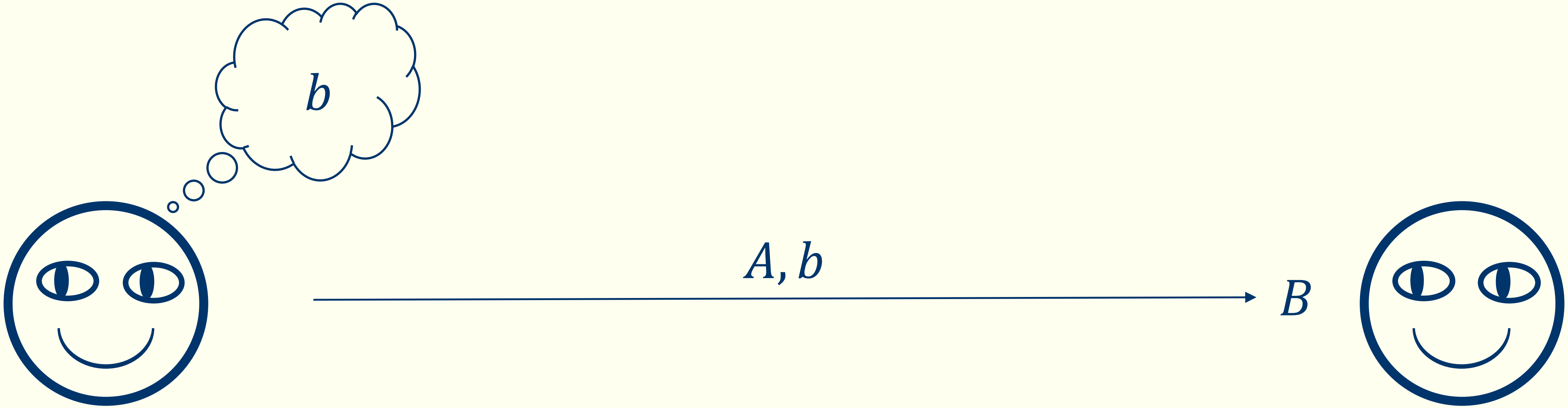

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- (Binding) For all (possibly inefficient) adversaries A,

$$\mathrm{F}\left(\mathrm{A}\left(\rho_A^{(0)}\right), \rho_A^{(1)}\right) \leq \mathrm{negl}(\lambda).$$

EFI pairs

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- Statistically Far: Binding gives us that the two states have high trace distance.
- Computationally Indistinguishable: Hiding guarantees that no efficient adversary can distinguish them.

The common Haar random state model

In the common Haar random state model (CHRS) [CCS'24, AGL'24], there are a collection of states $\{|\psi_\ell\rangle\}_{\ell \in \mathbb{N}}$ that are sampled uniformly at random from the Haar measure, and all parties get sample access to the states.

Our main result

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Since quantum bit commitments exist in the common Haar random state model [CCS’24, AGL’24], this separates quantum bit commitments and one-way state generators.

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- If all accept, halt and output k'.
- Otherwise, uncompute every $\mathsf{Ver}(k', \rho_k)$, and continue.

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Using a pseudo-random generator (against PSPACE) and deferred measurement, the entire algorithm can be described by a deterministic PSPACE quantum circuit.

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Un-entangled versus entangled?

Landscape of quantum cryptography now