Separating QMA from QCMA with a classical oracle

John Bostanci, Jonas Haferkamp, Chinmay Nirkhe, and Mark Zhandry

Chicago junior theorists workshop, 2025

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In complexity theory, the class NP captures the kinds of problems that we hope to be able to prove to one another.

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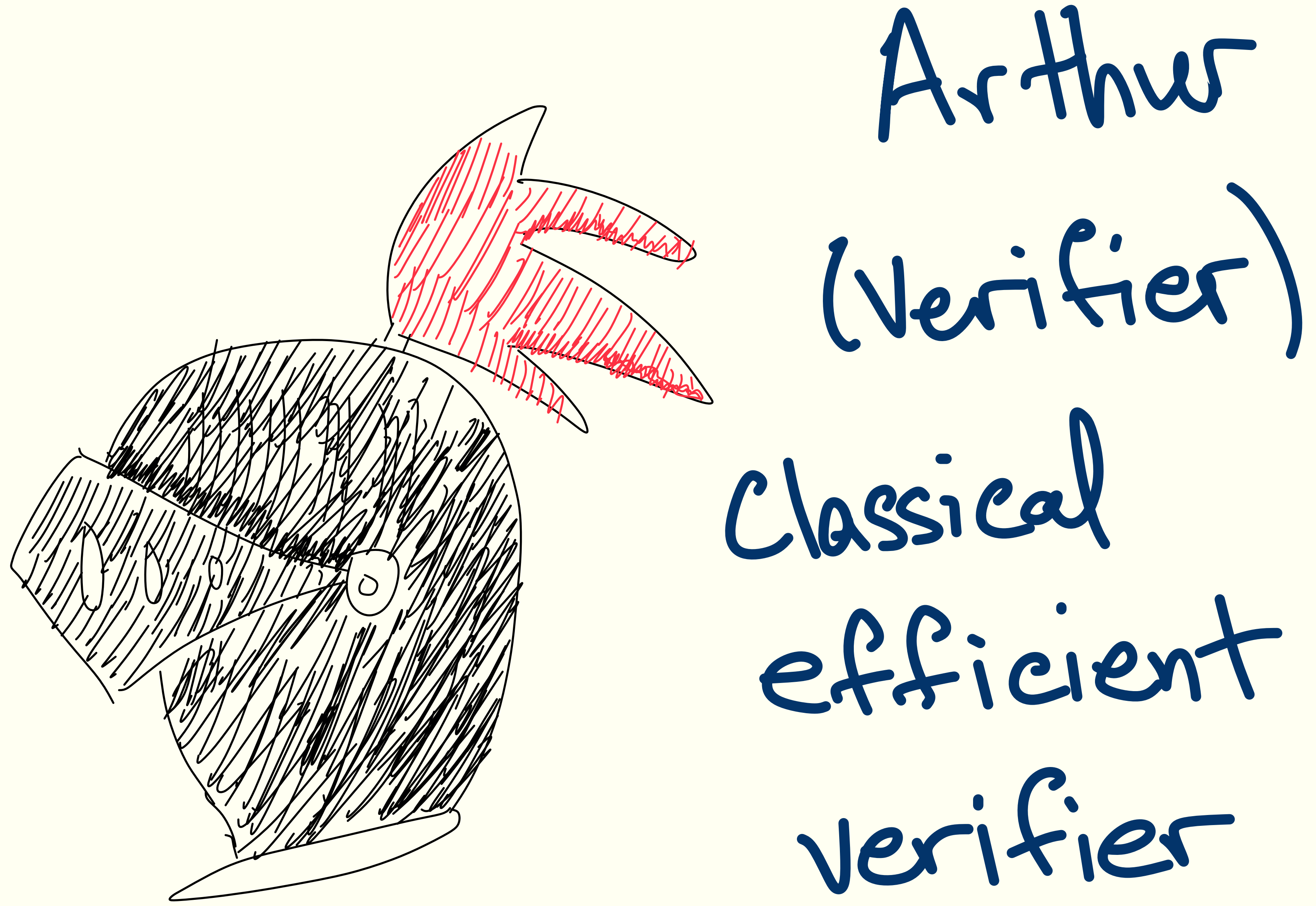

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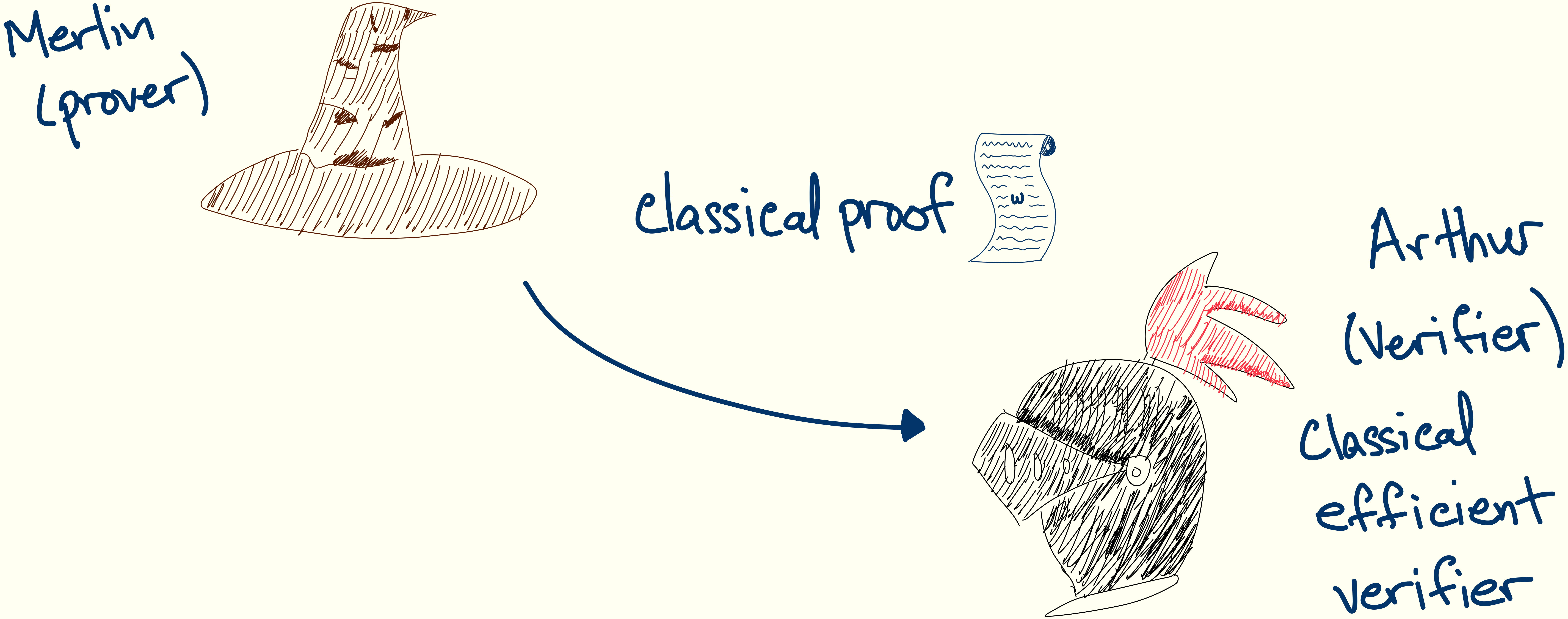

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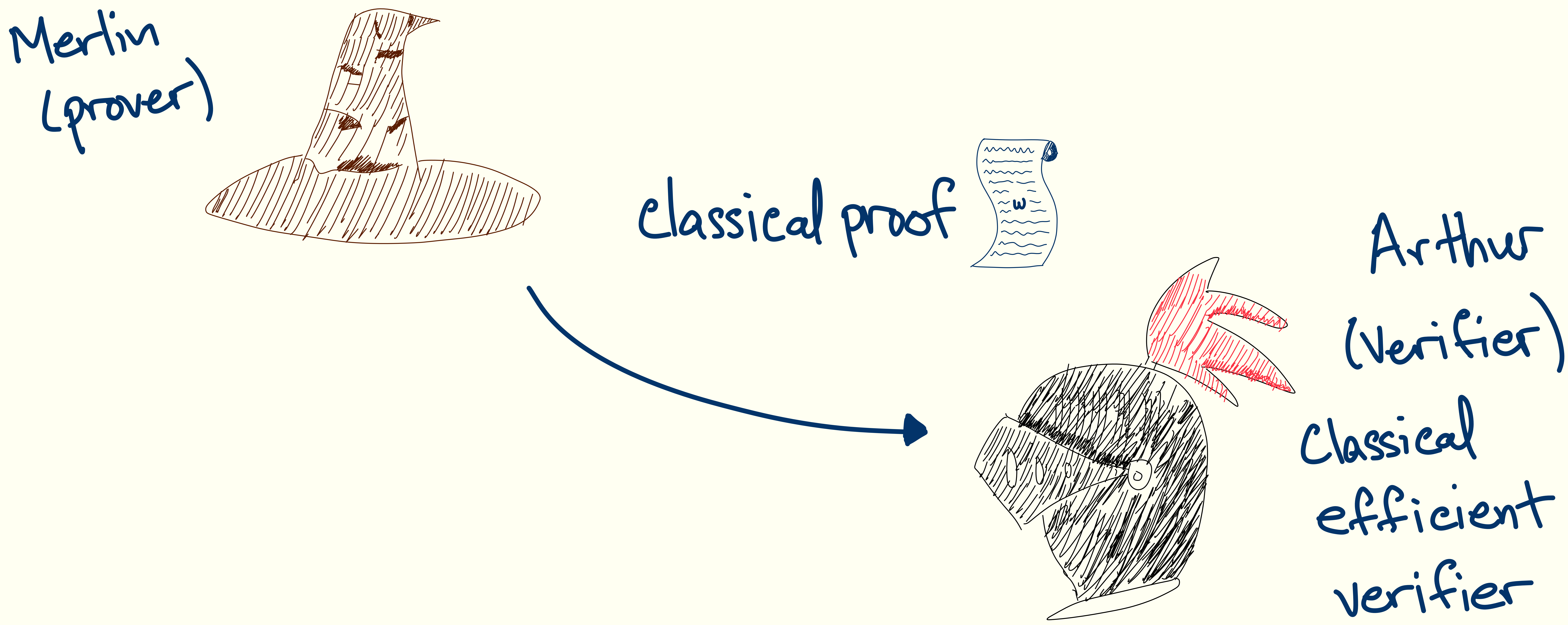

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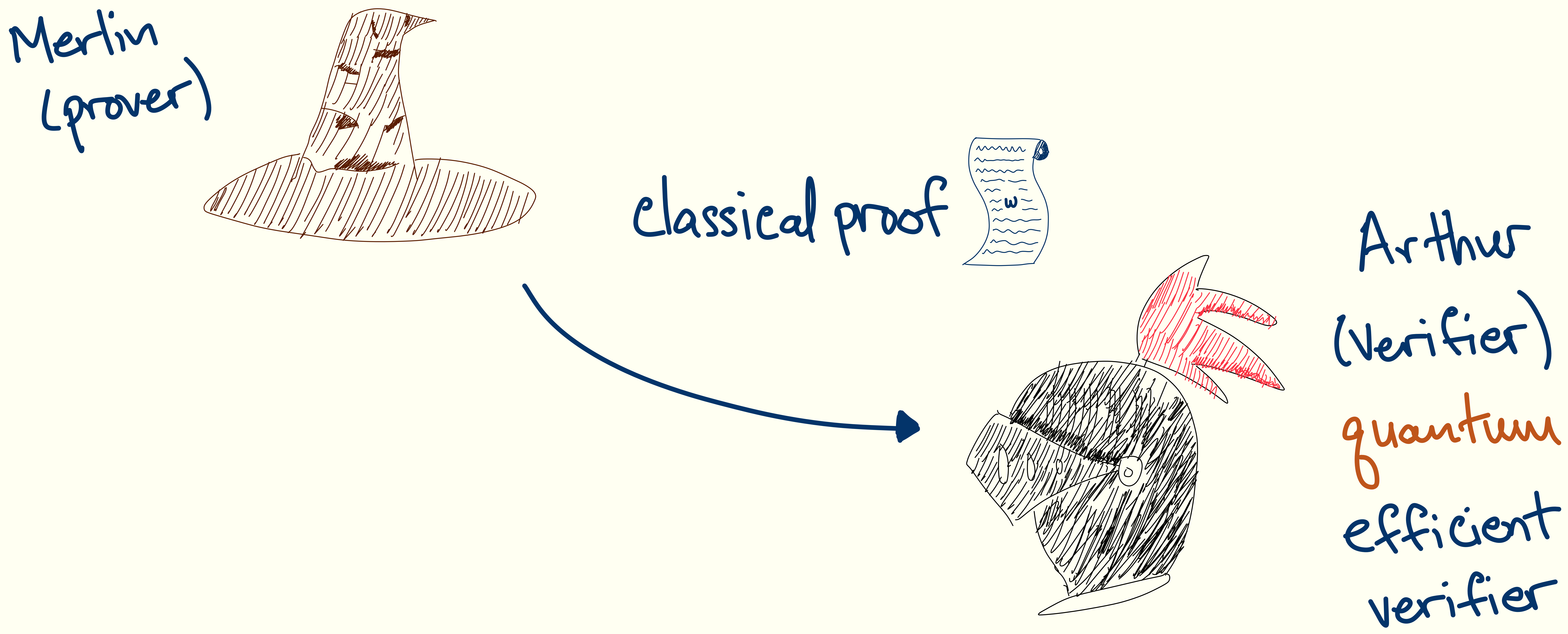
With quantum computers, we can compare the relative powers of quantum proofs and classical proofs. QCMA captures the kinds of problems we could prove classically.



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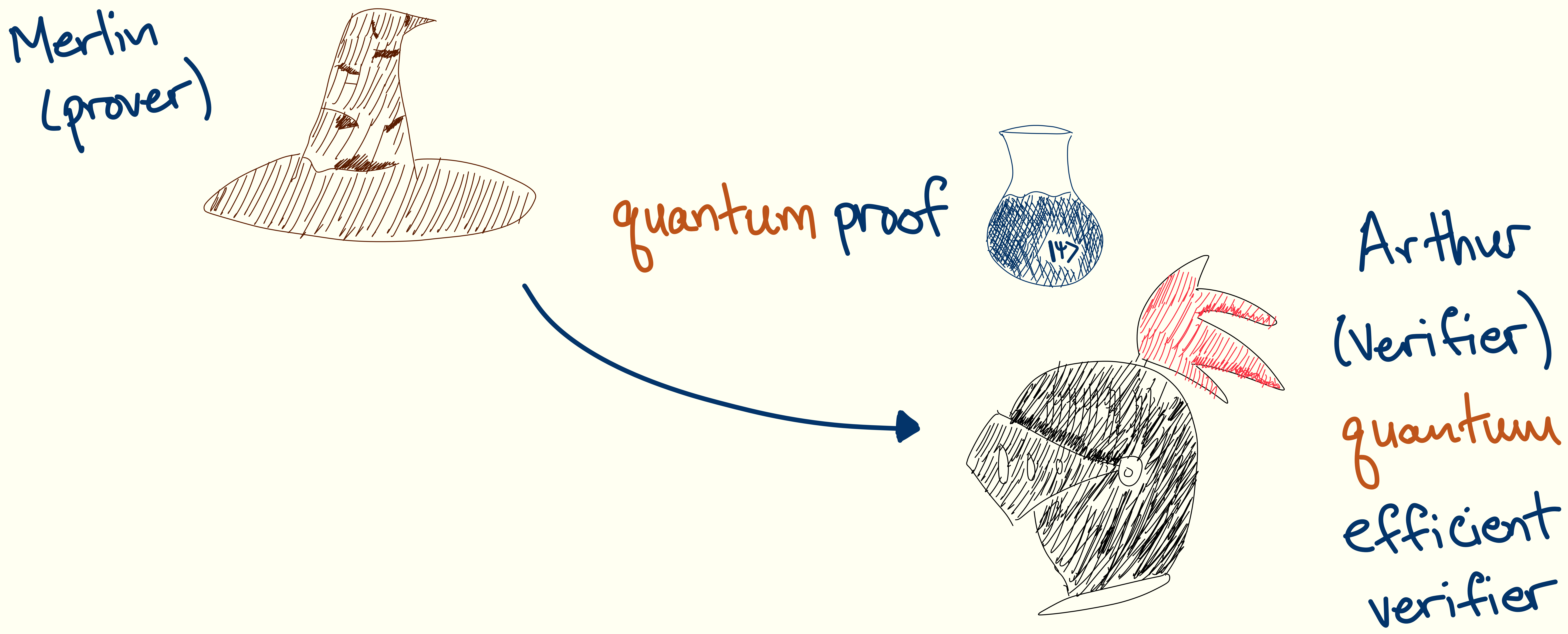
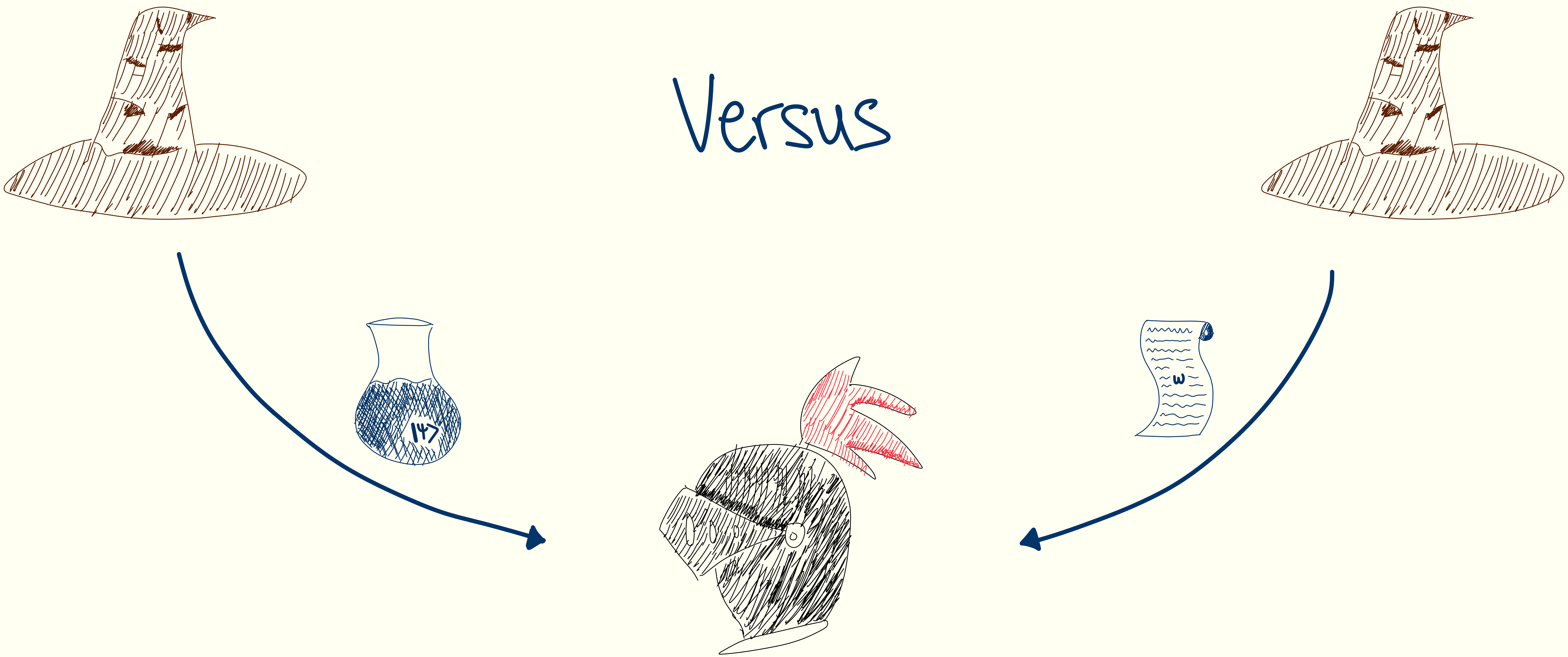


Is QMA = QCMA? [AN'02]



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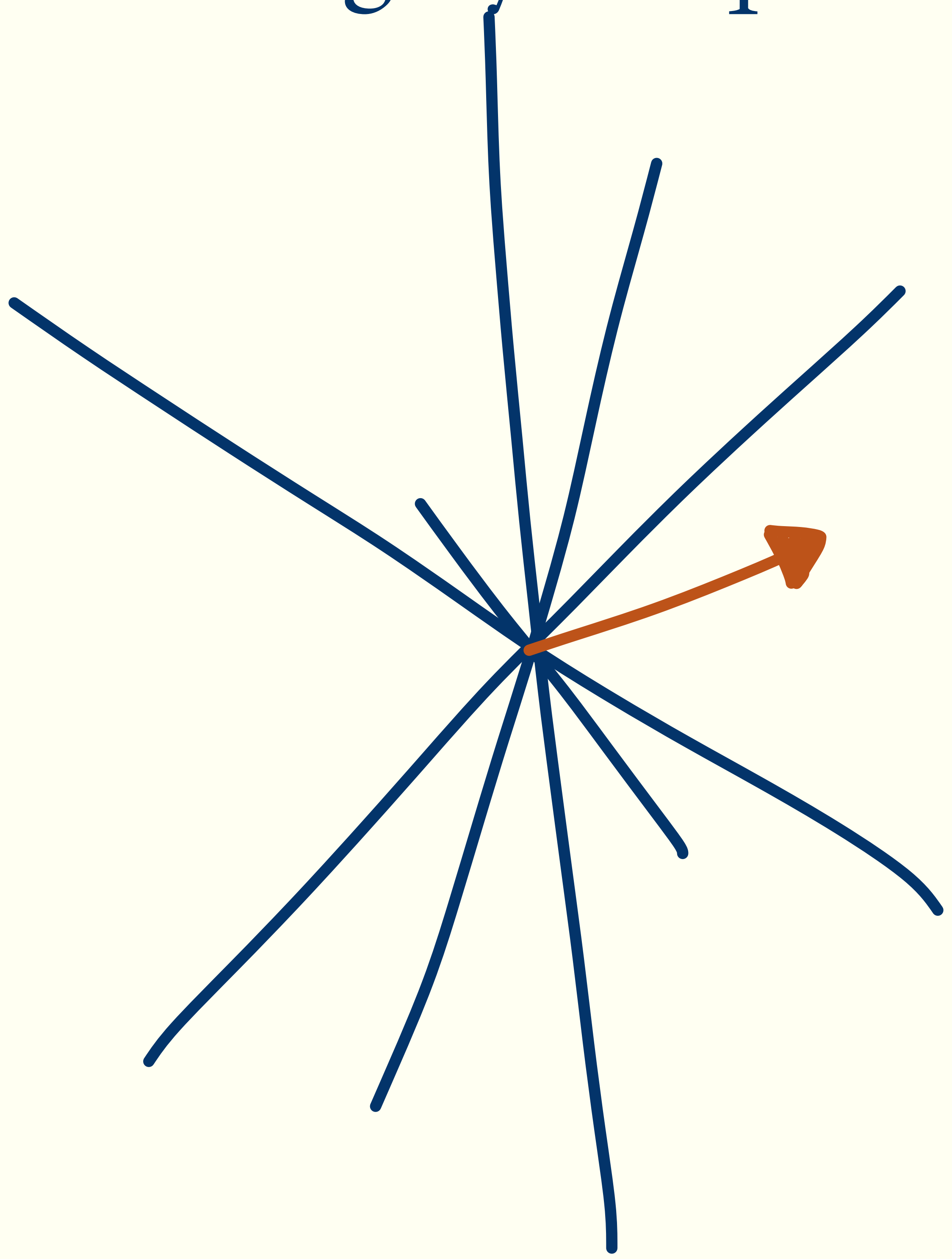
If QCMA = QMA, then anything you could verify about a quantum state could be written down as a classical string!

Otherwise, there must be something interesting you could verify about a quantum state that you can only learn from having a copy of the state!

A quantum oracle separation [AK'06]

Recall: A quantum state on n-qubits is a vector of 2^n complex numbers
→ there are roughly 2^{2^n} quantum states on n-qubits.



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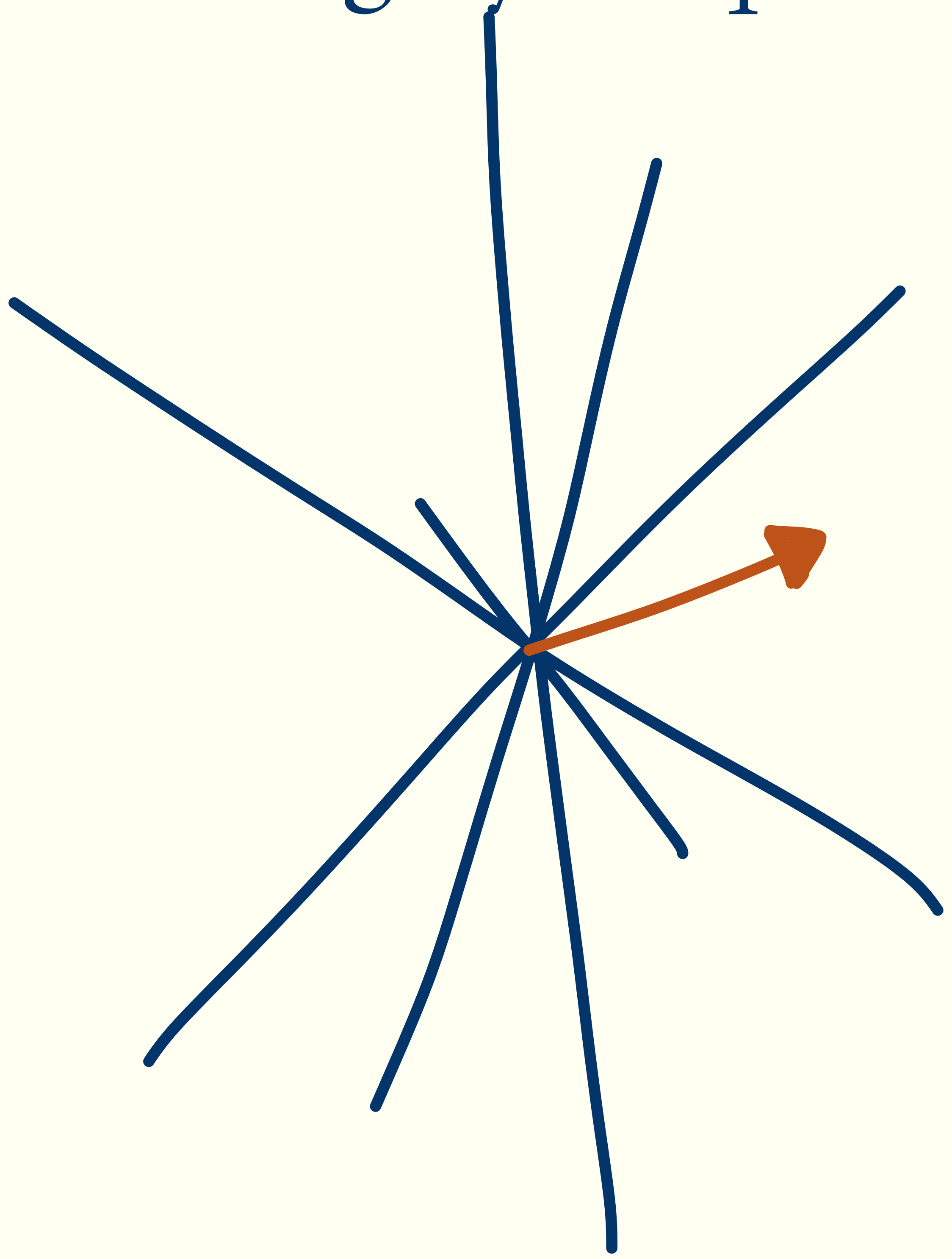

$$|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha_x |x\rangle$$

Concentration: Any collection of $2^{\mathrm{poly}(n)}$ states will fail to be close to a random state!

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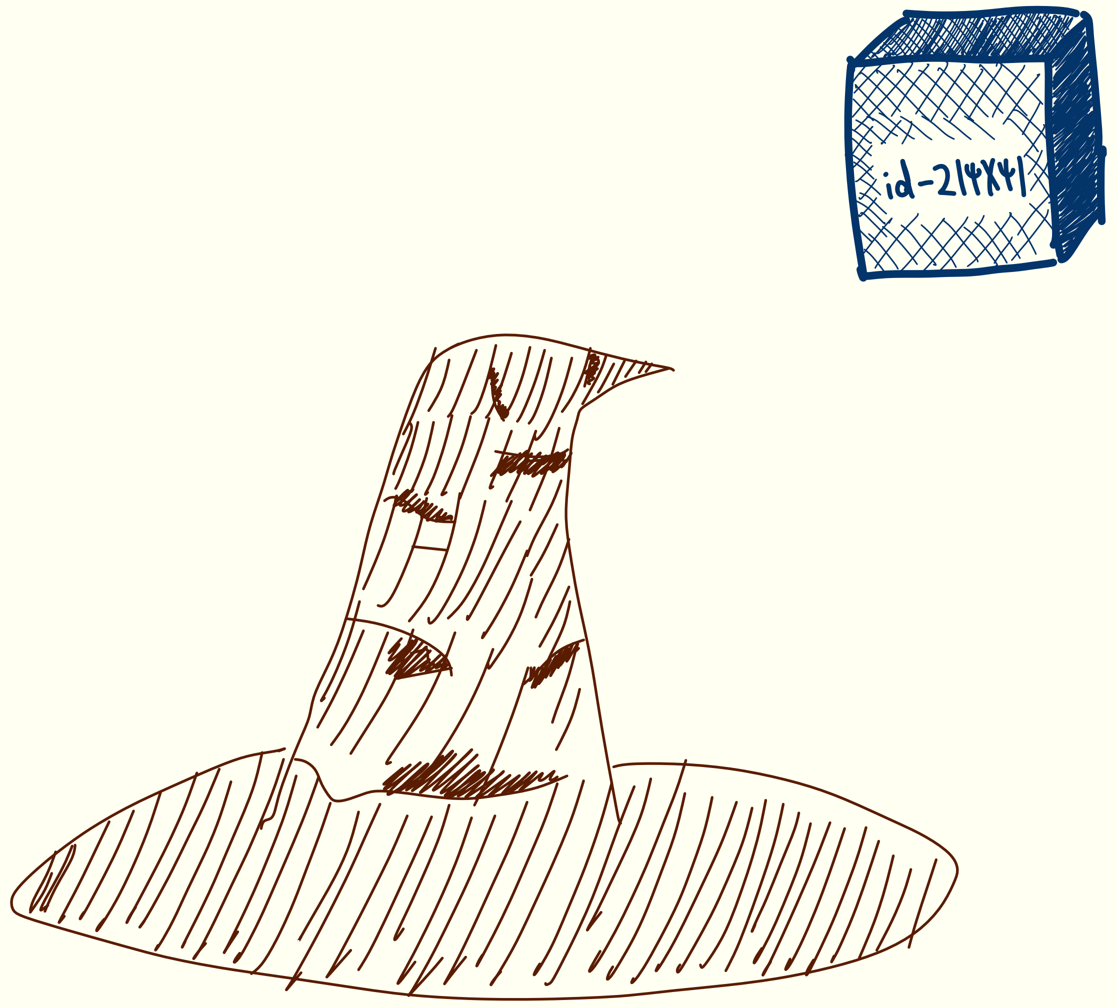
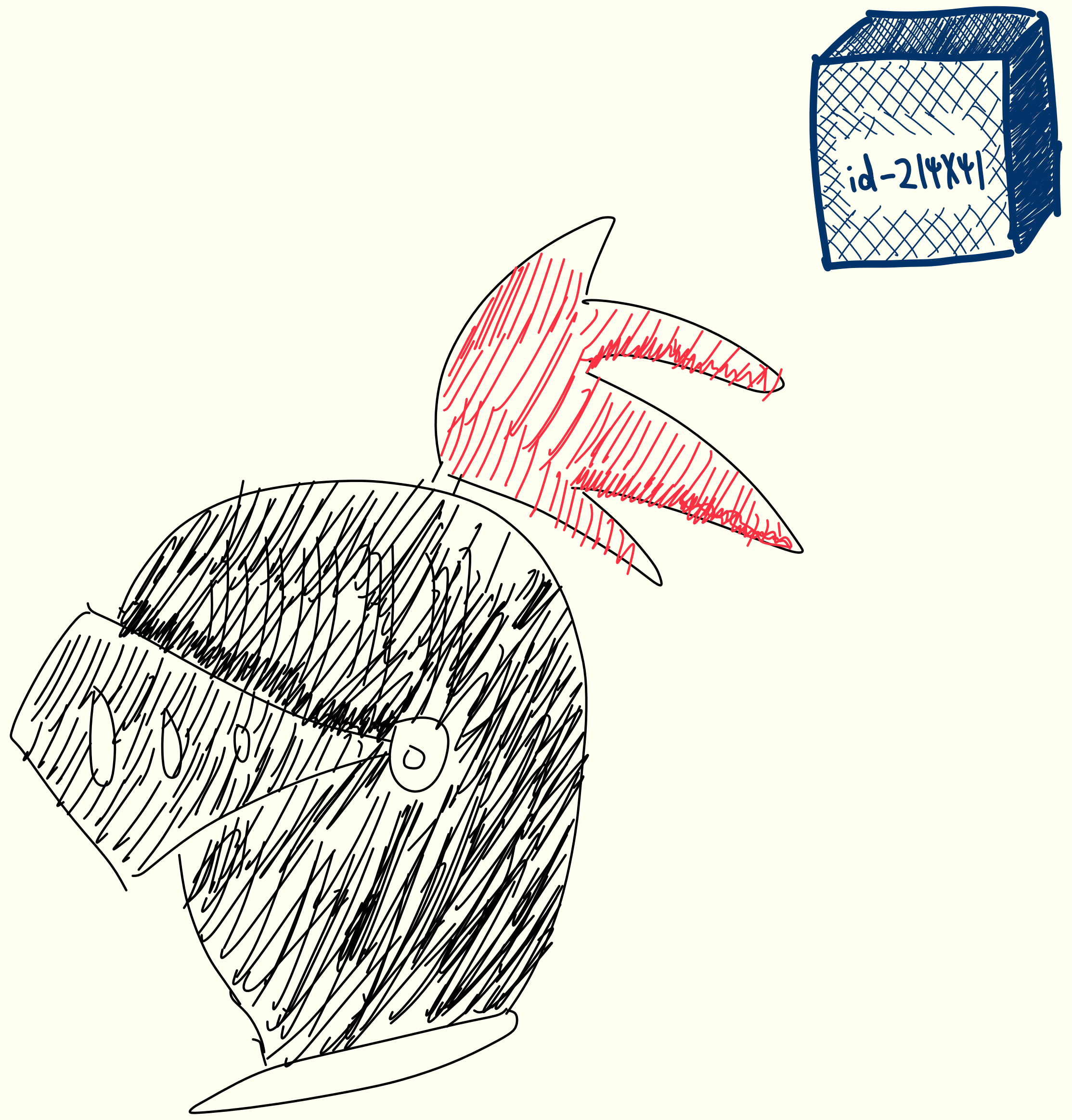
Claim: Oracle that accepts a random state (or nothing) separates QMA from QCMA.





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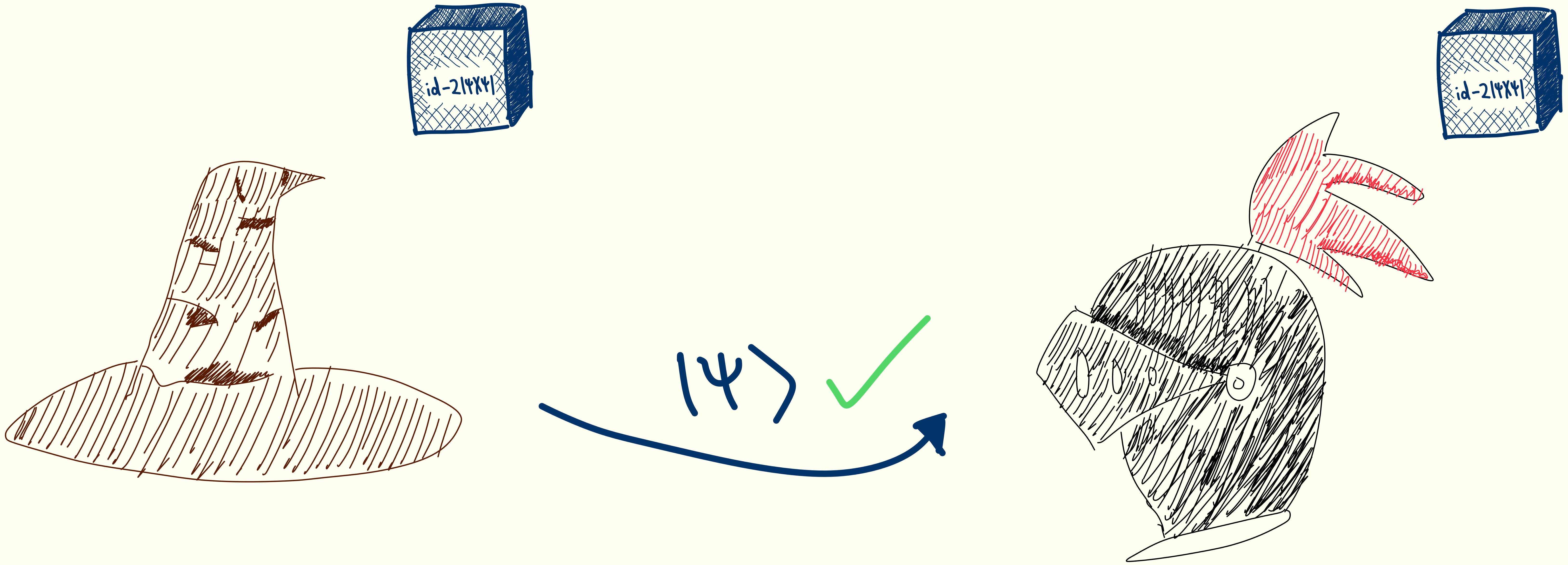

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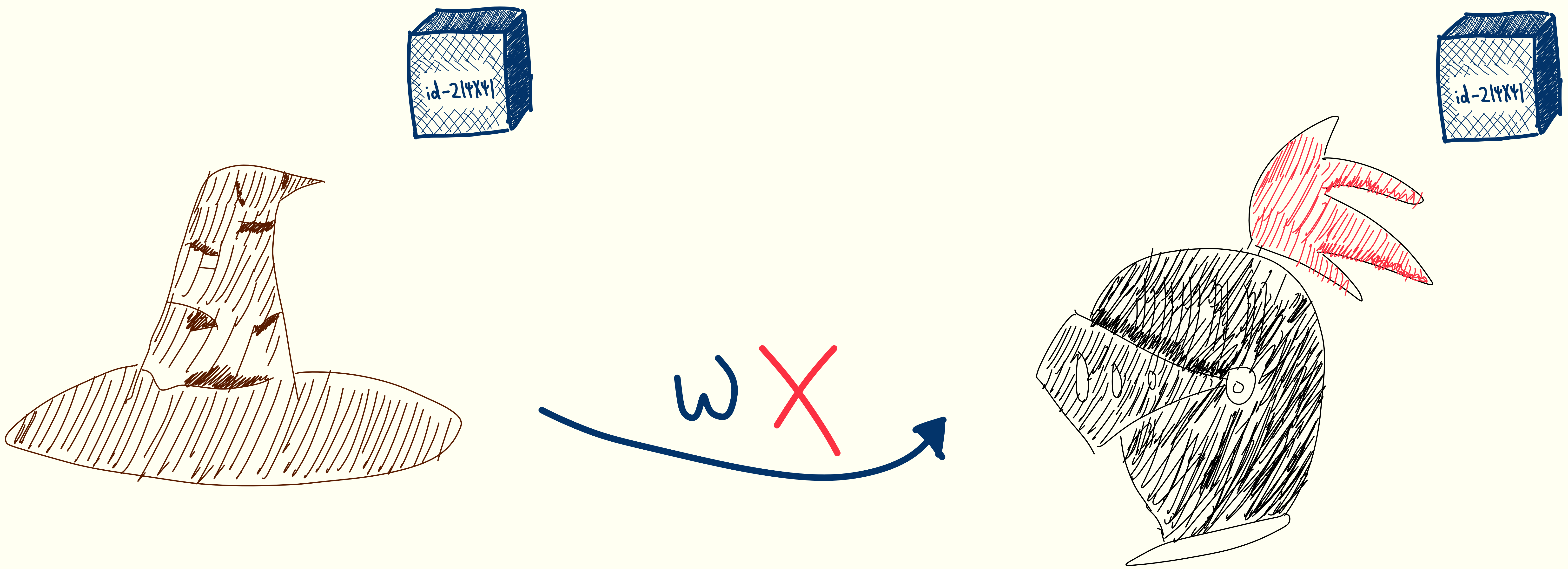
→ Any QCMA verifier will only be able to check $2^{\text{poly}(n)}$ different quantum states. For almost all random states, it won't be able to check.



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But for any "classicalization" of this oracle, classical witness can now actually say something about the oracle, so the problem gets a lot more challenging!

History of the QMA **versus** QCMA **problem**

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Fefferman & Kimmel '15: In-place permutation oracle. Problem corresponds to set size estimation.

Natarajan & Nirkhe '22: Distribution testing oracle. Problem corresponds to size estimation of an expander graph.

Li, Liu, Pelecanos, Yamakawa '23: Separation assuming only classical queries. Based on "Verifiable Quantum Advantage without Structure".

'02 —————————————————————— '25

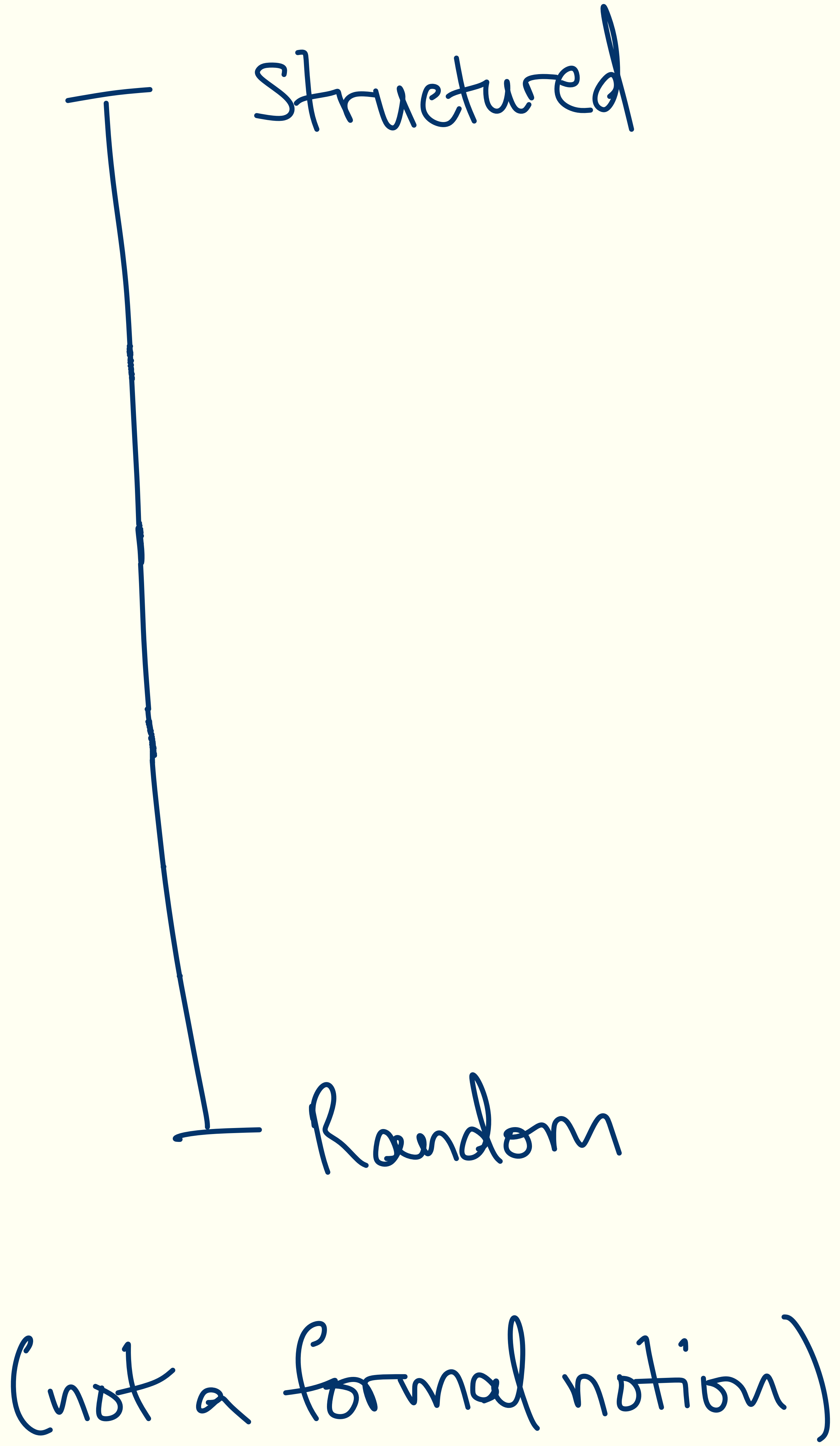
Lutomirski '11: Proposed the expander mixing problem as a candidate classical oracle separation.

Ben-David & Kundu '24: Bounded adaptivity, based on "Verifiable Quantum Advantage without Structure".

Liu, Mutreja, Yuen '25: Aaronson-Ambainis-like conjecture implies QMA QCMA separation. Problem corresponds to size estimation of an expander graph.

Why is this problem so hard?

To me, the problem has been "stuck" in between two competing desires for a while.



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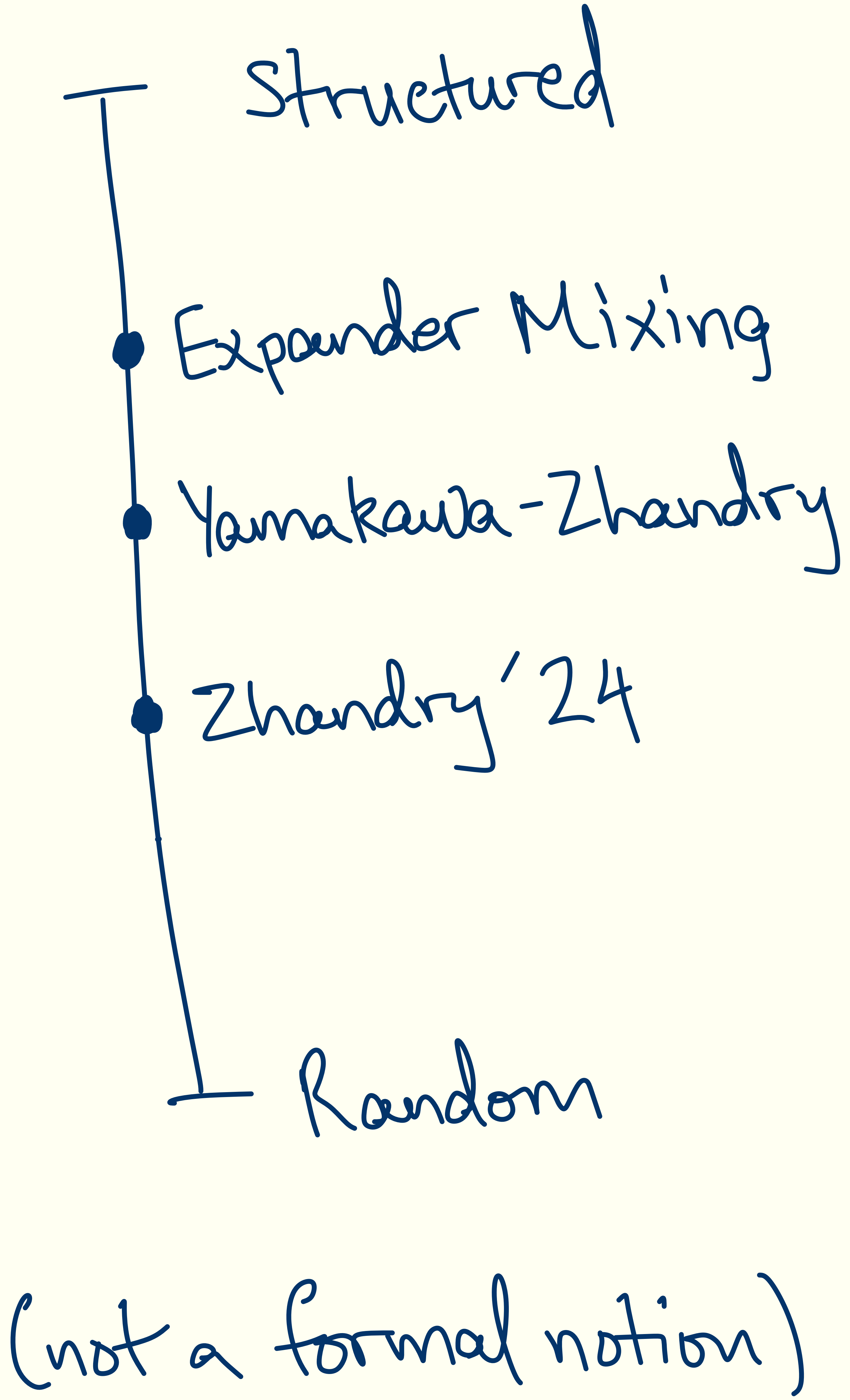

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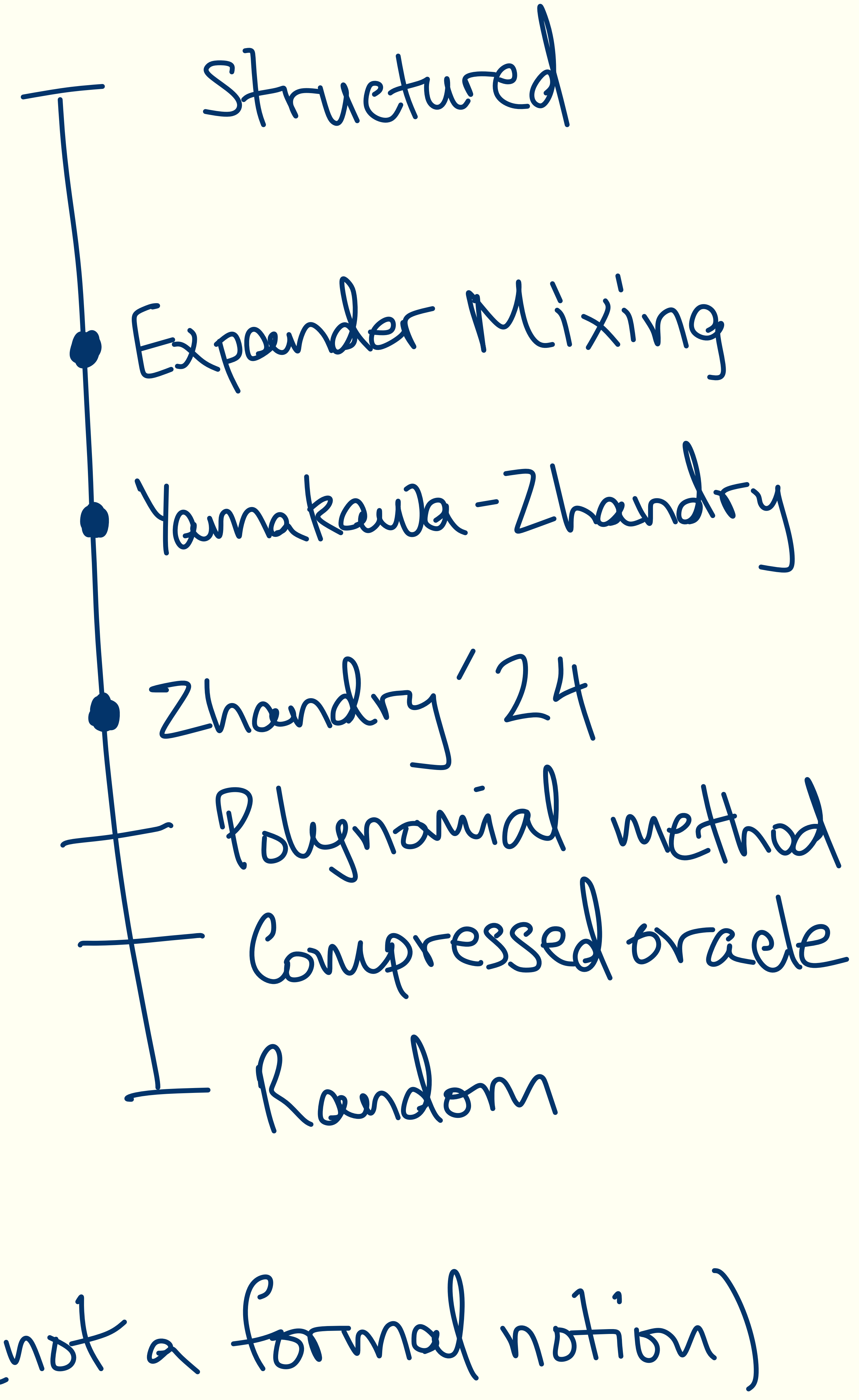
- But, quantum lower bound techniques usually take advantage of the randomness of the oracle.
  → Need a new technique for analyzing some kind of structured classical oracles.



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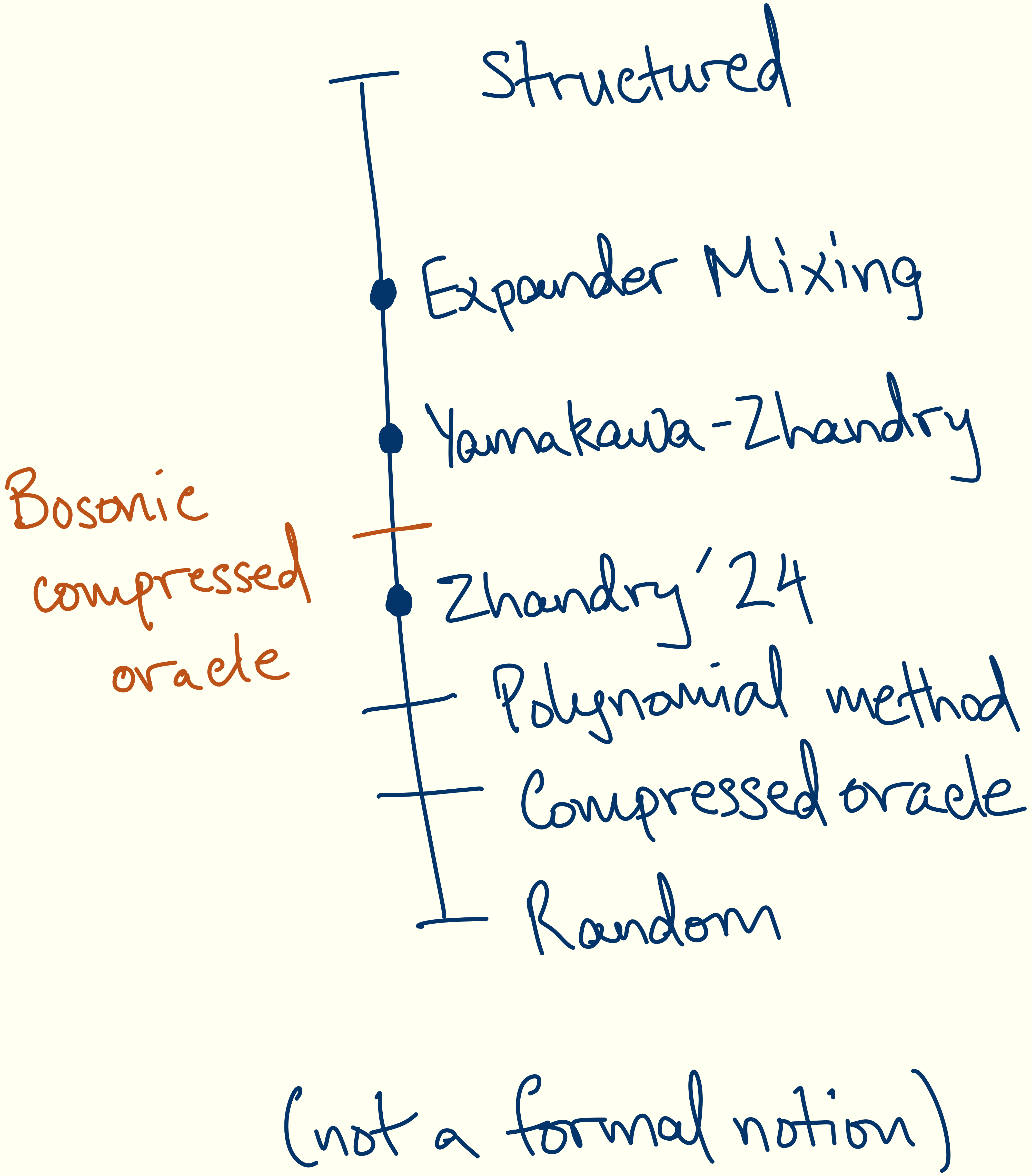
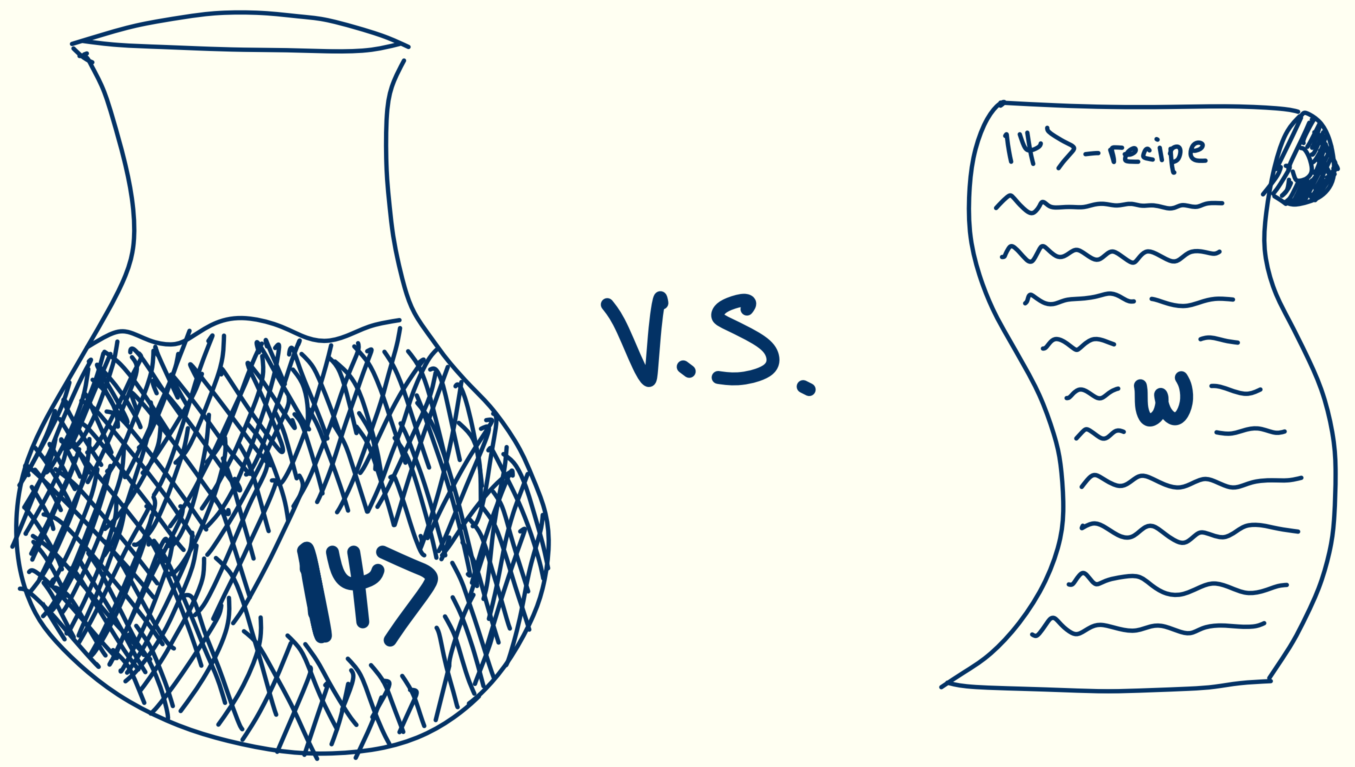
Our paper bridges the gap, taking the less structured oracle of Zhandry’24, and introducing new analysis to understand Fourier-related sets.



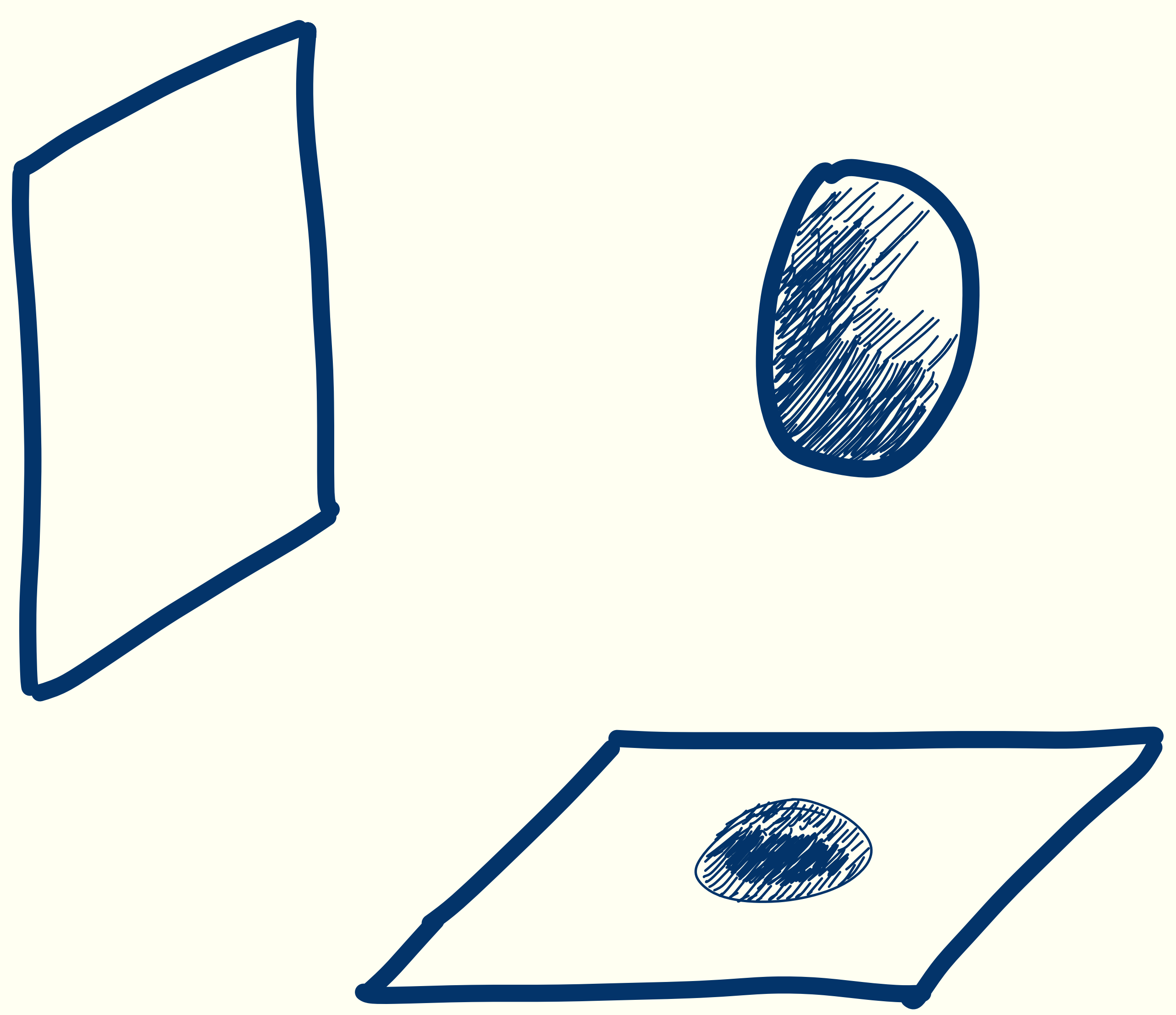
We prove that there is a classical oracle relative to which QMA $\neq$ QCMA



Computing in two bases

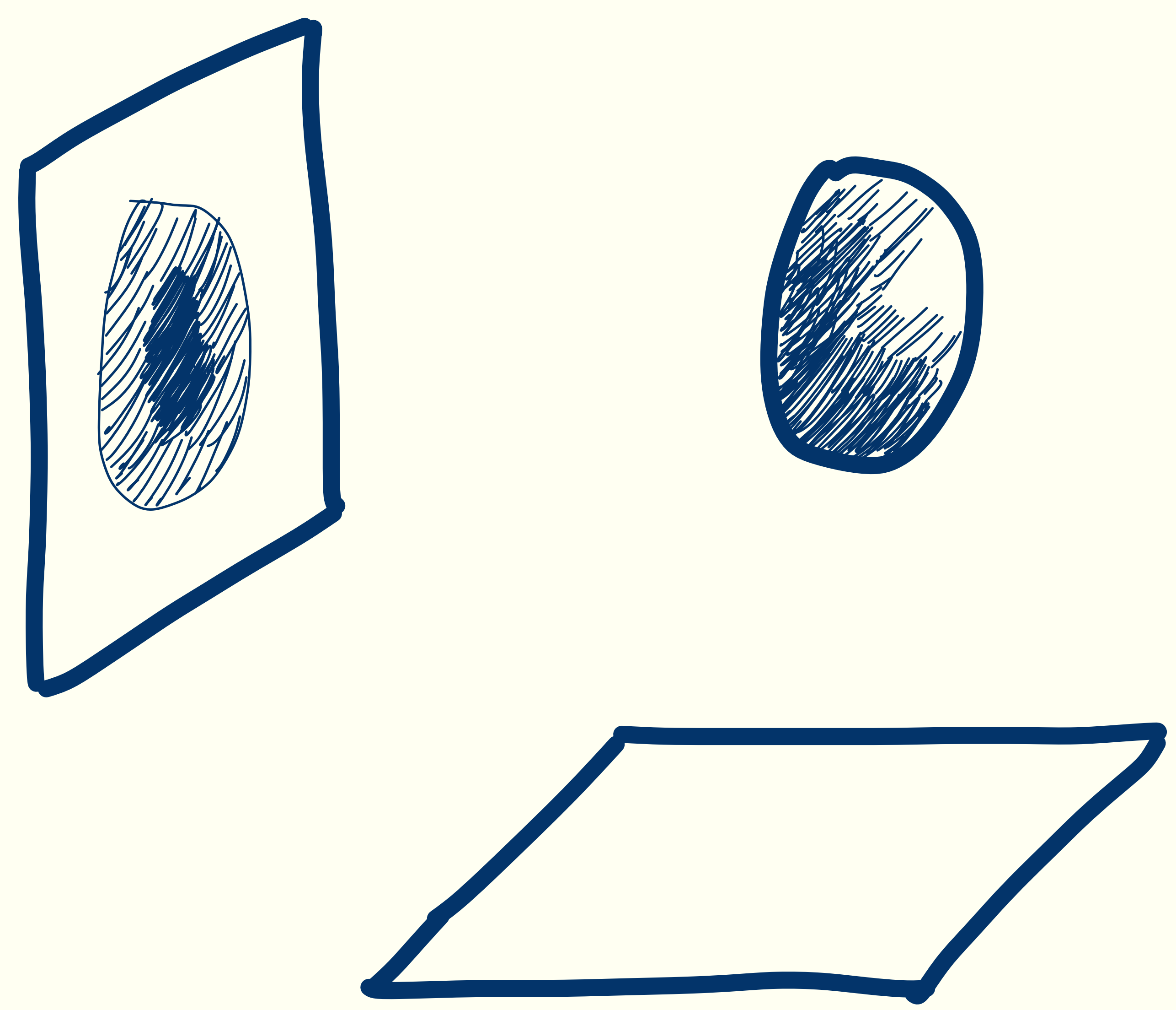
Quantum states can be viewed from one of two bases:

Position (Standard)



&

Momentum (Hadamard)



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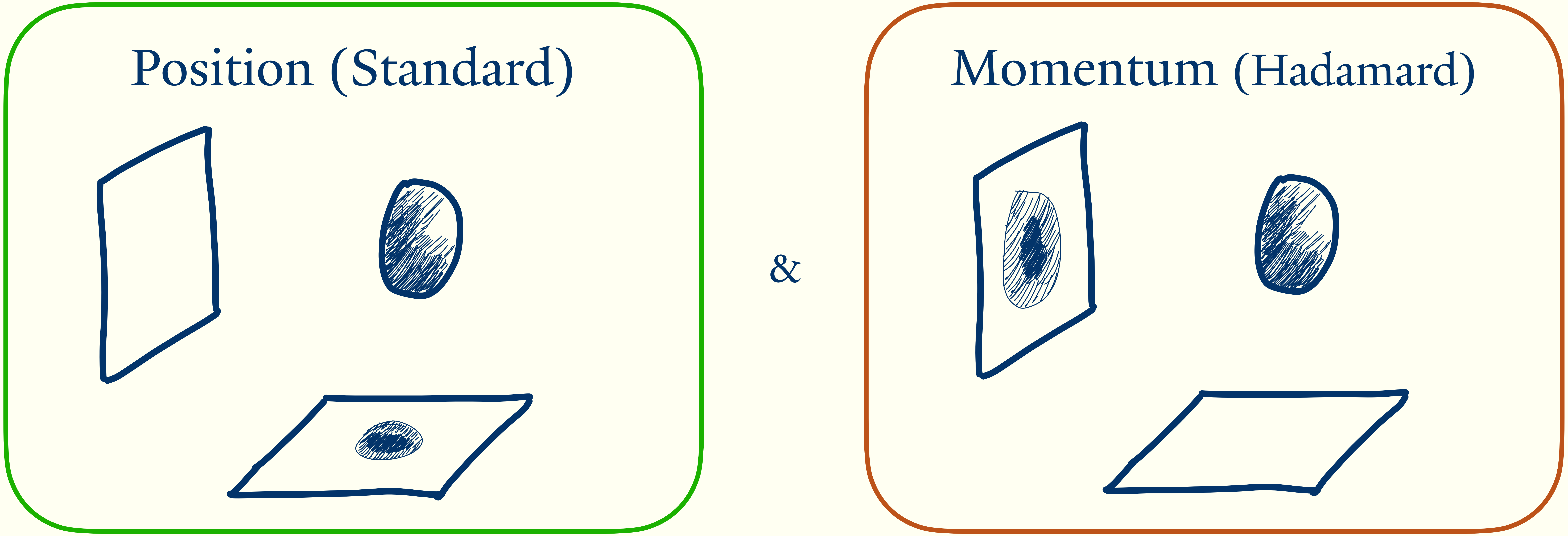
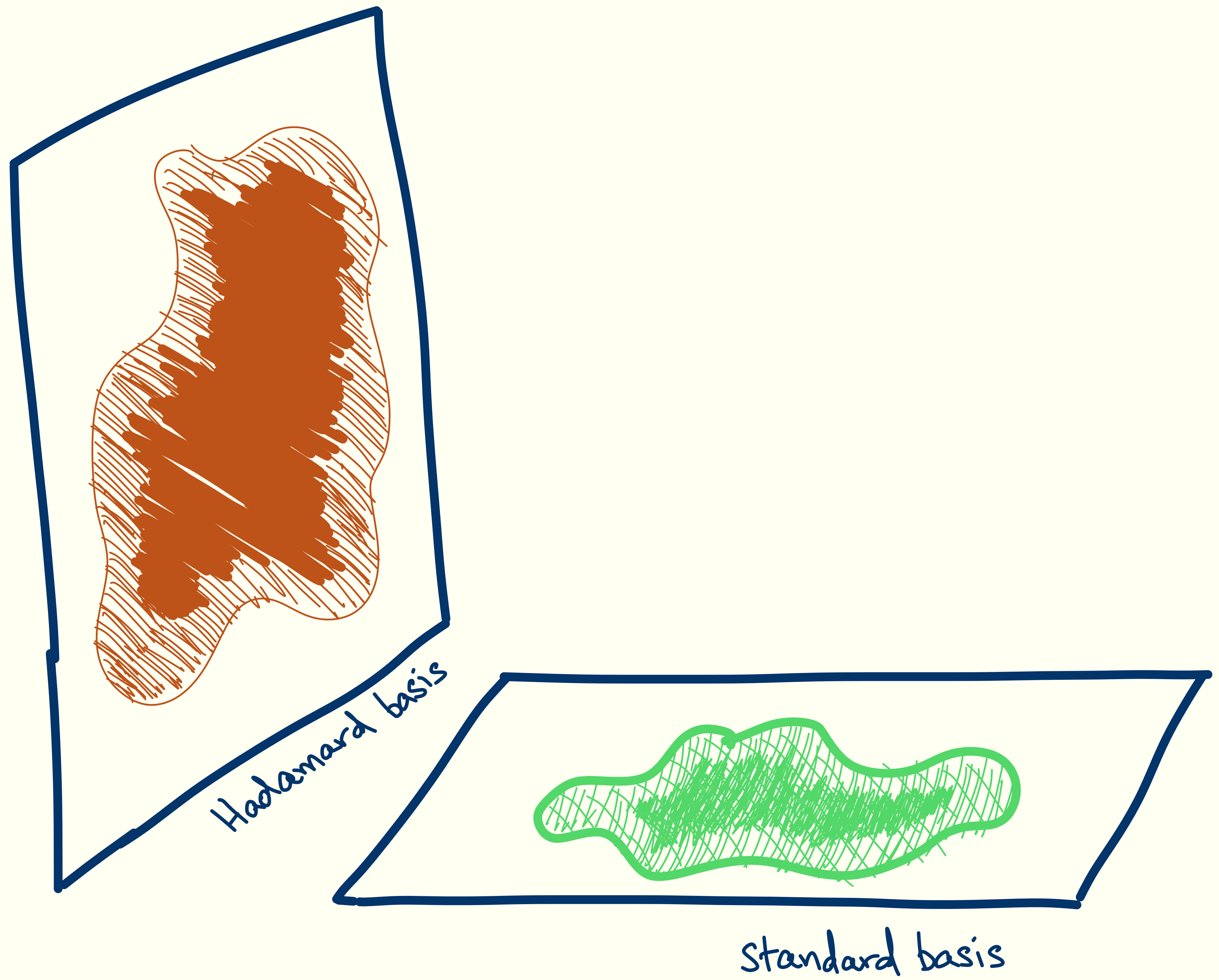


Rough intuition: The "size" of the shadows in the standard and Hadamard basis should multiply to a fixed number for all n-qubit states ($\sim 2^n$).

The spectral Forrelation problem

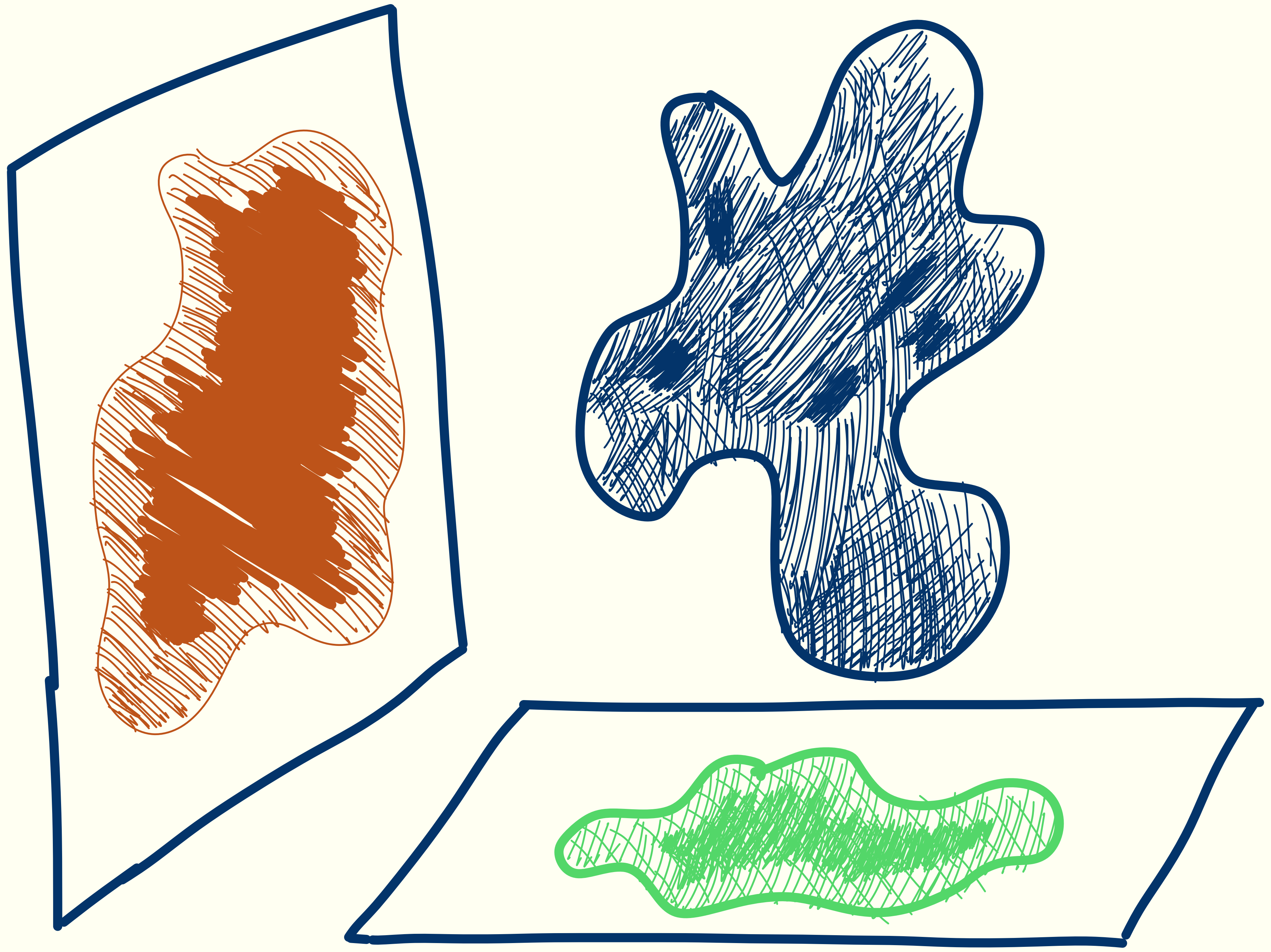
The spectral Forrelation problem is a problem about pairs of sets (S, U), which we treat as oracles through the set membership functions. S~positions, and U~momentums.



The spectral Forrelation problem

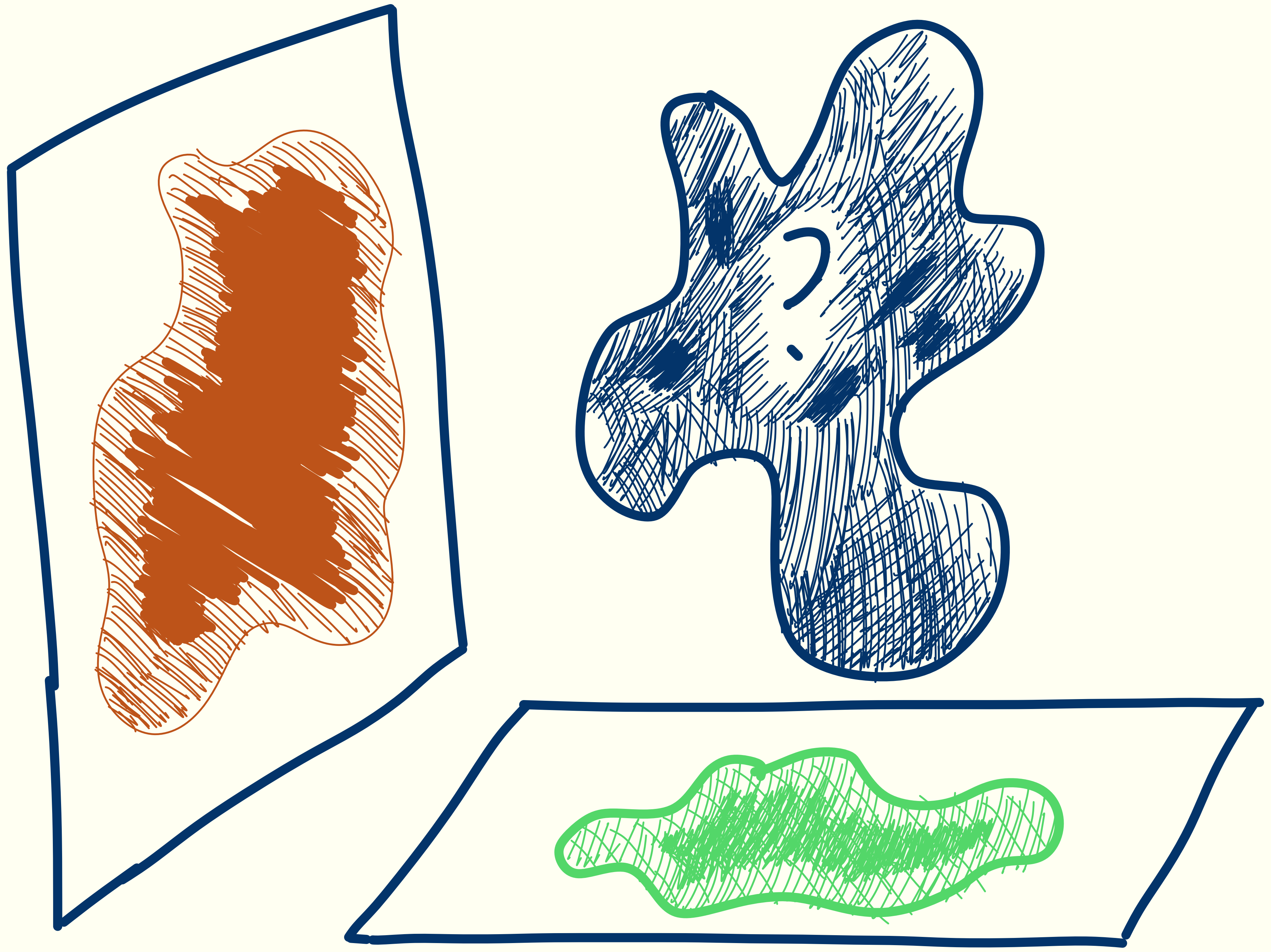
We say that two sets (S, U) are α-spectrally Forrelated if there is a state $|\psi\rangle$ such that

$$\|\Pi_U \cdot H^{\otimes n} \cdot \Pi_S |\psi\rangle\|^2 \geq \alpha$$



The spectral Forrelation problem

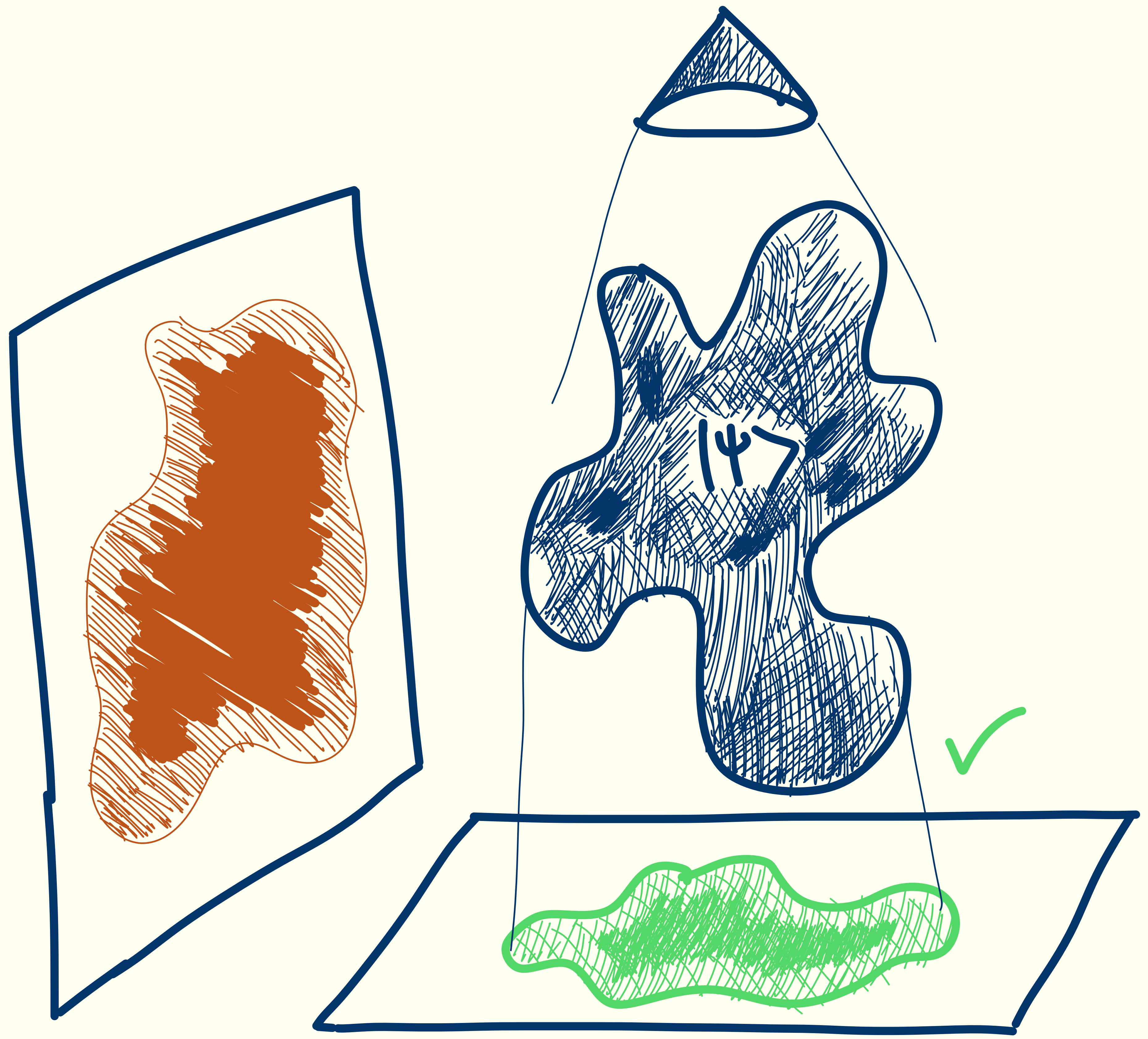
Given oracle access to two sets (S, U) (via set membership functions), determine if there is a state $|\psi\rangle$ such that $\|\Pi_U \cdot H^{\otimes n} \cdot \Pi_S |\psi\rangle\|^2$ is large ($\geq 59/100$) or small ($\leq 57/100$), promised that one of the two is the case.



Spectral Forrelation is in QMA

Given a copy of a state $|\psi\rangle$:

- Use S oracle to measure the POVM $\{\Pi_S, \text{id} - \Pi_S\}$, reject if the outcome is $\text{id} - \Pi_S$.



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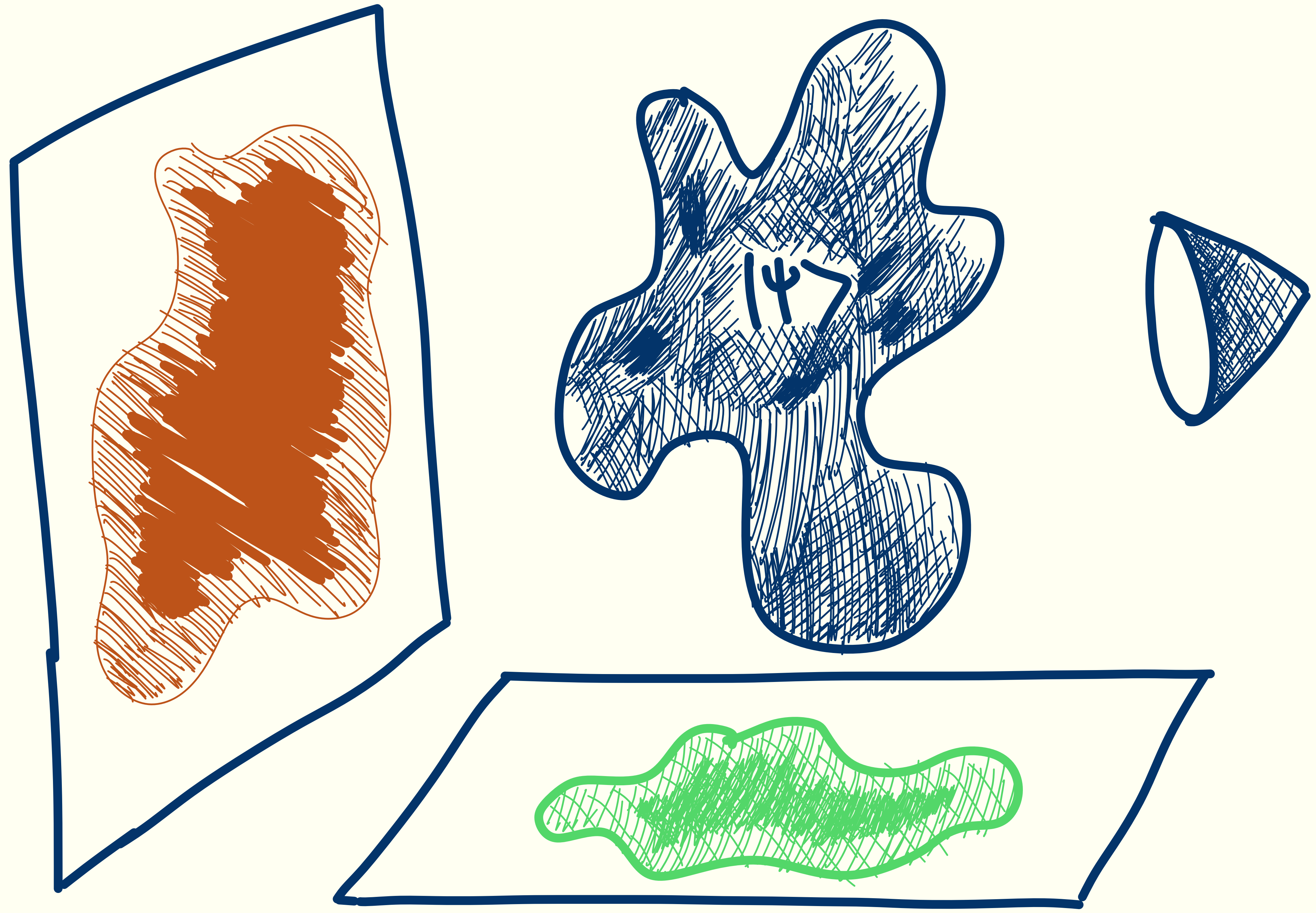

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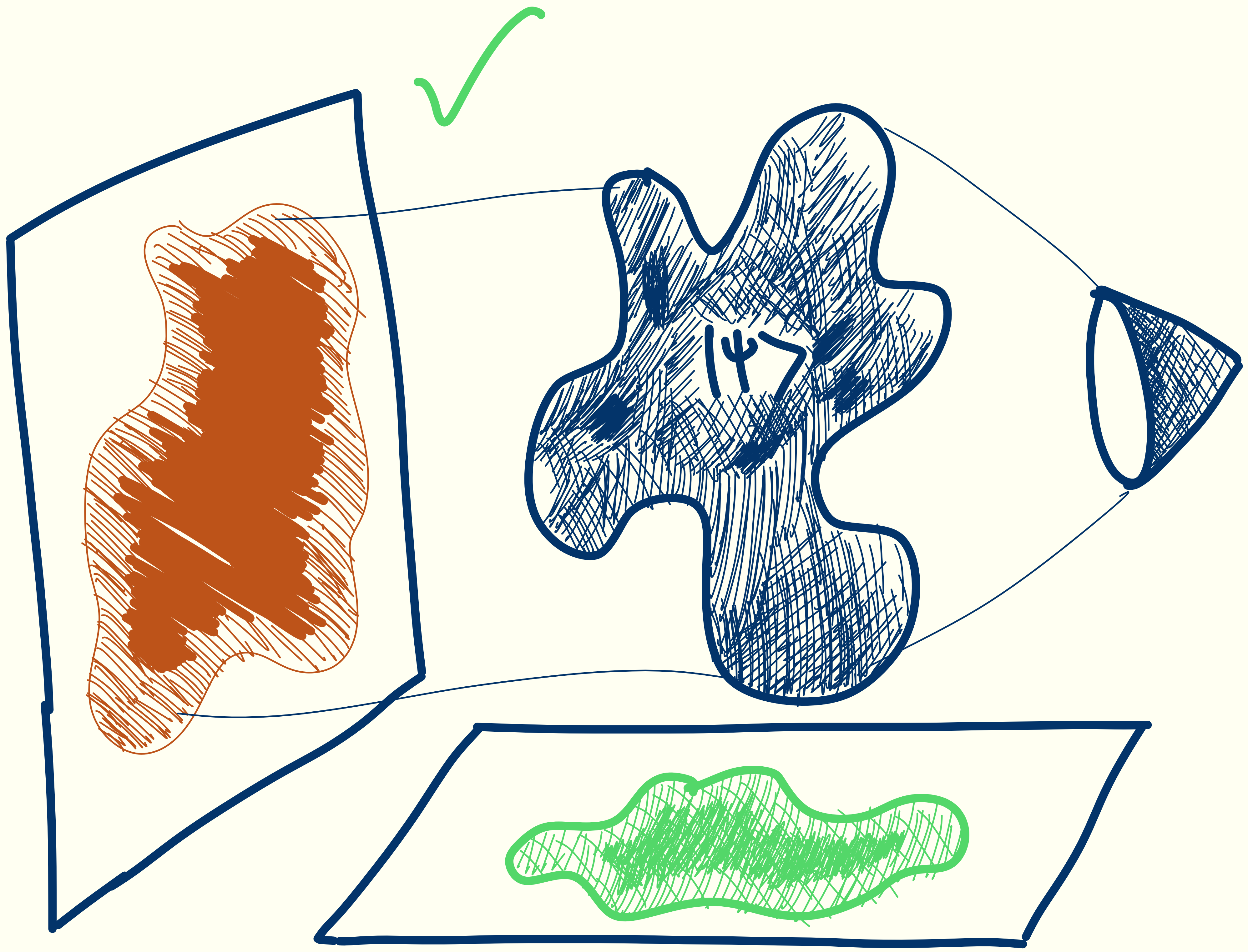
- Accept.



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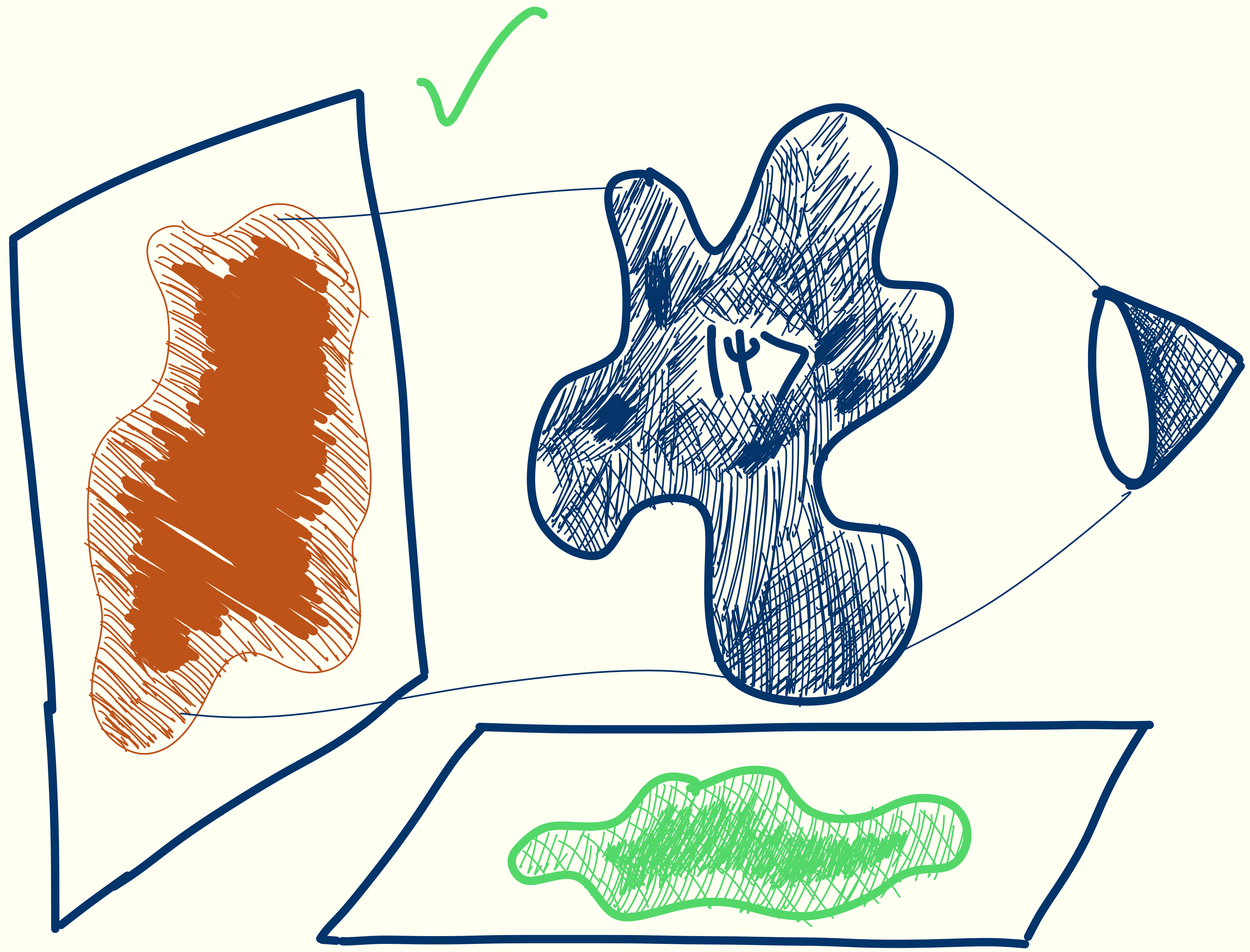
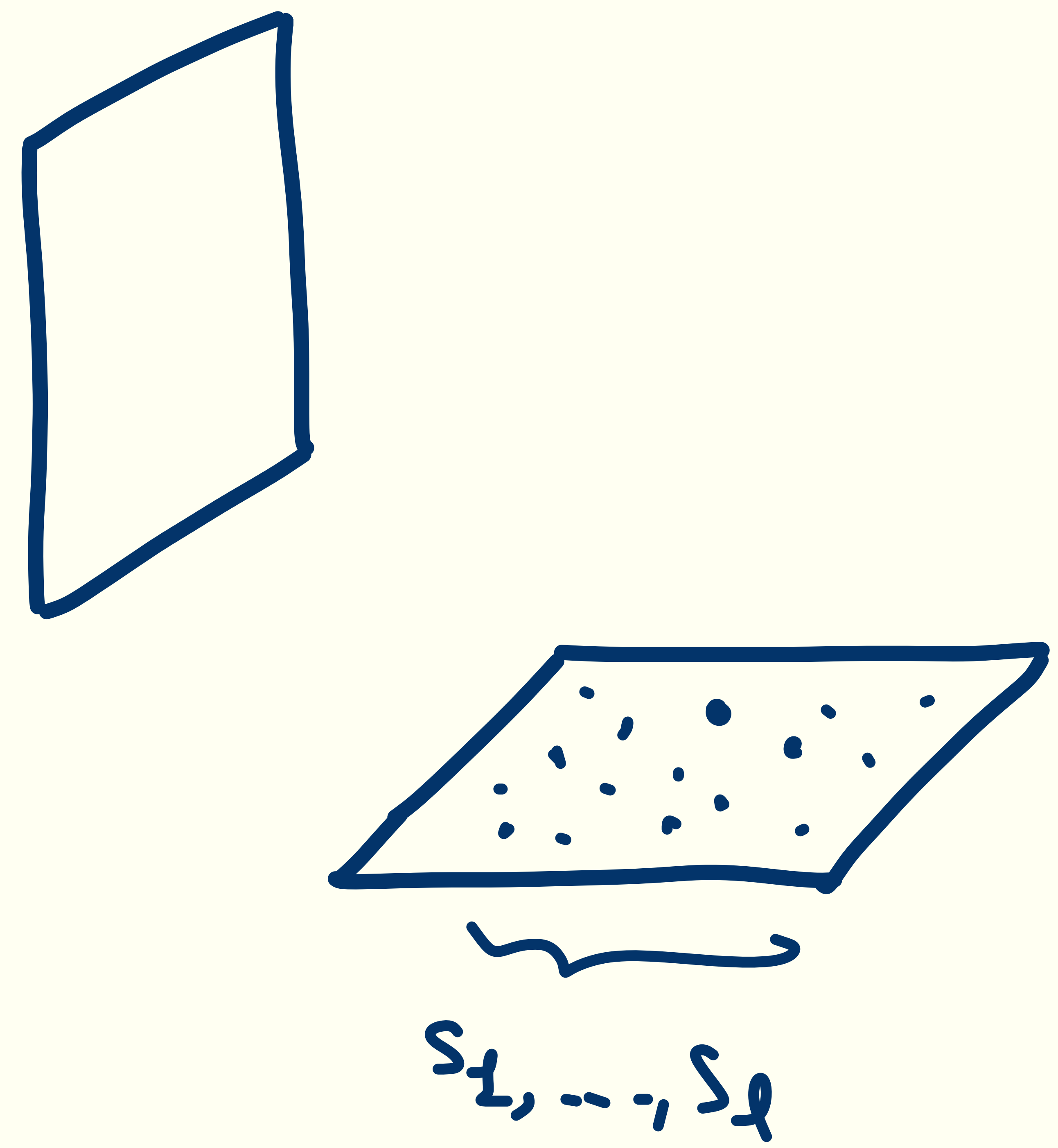


Claim: This verifier accepts with probability: $\| \Pi_U \cdot H^{\otimes n} \cdot \Pi_S |\psi\rangle \|^2$.
Sequential amplification can bring this to the standard 2/3 or 1/3.

Classicalizing a random quantum state

- We will first sample $\ell = 2^{n/10}$ many random elements $s_1, \ldots, s_\ell$.



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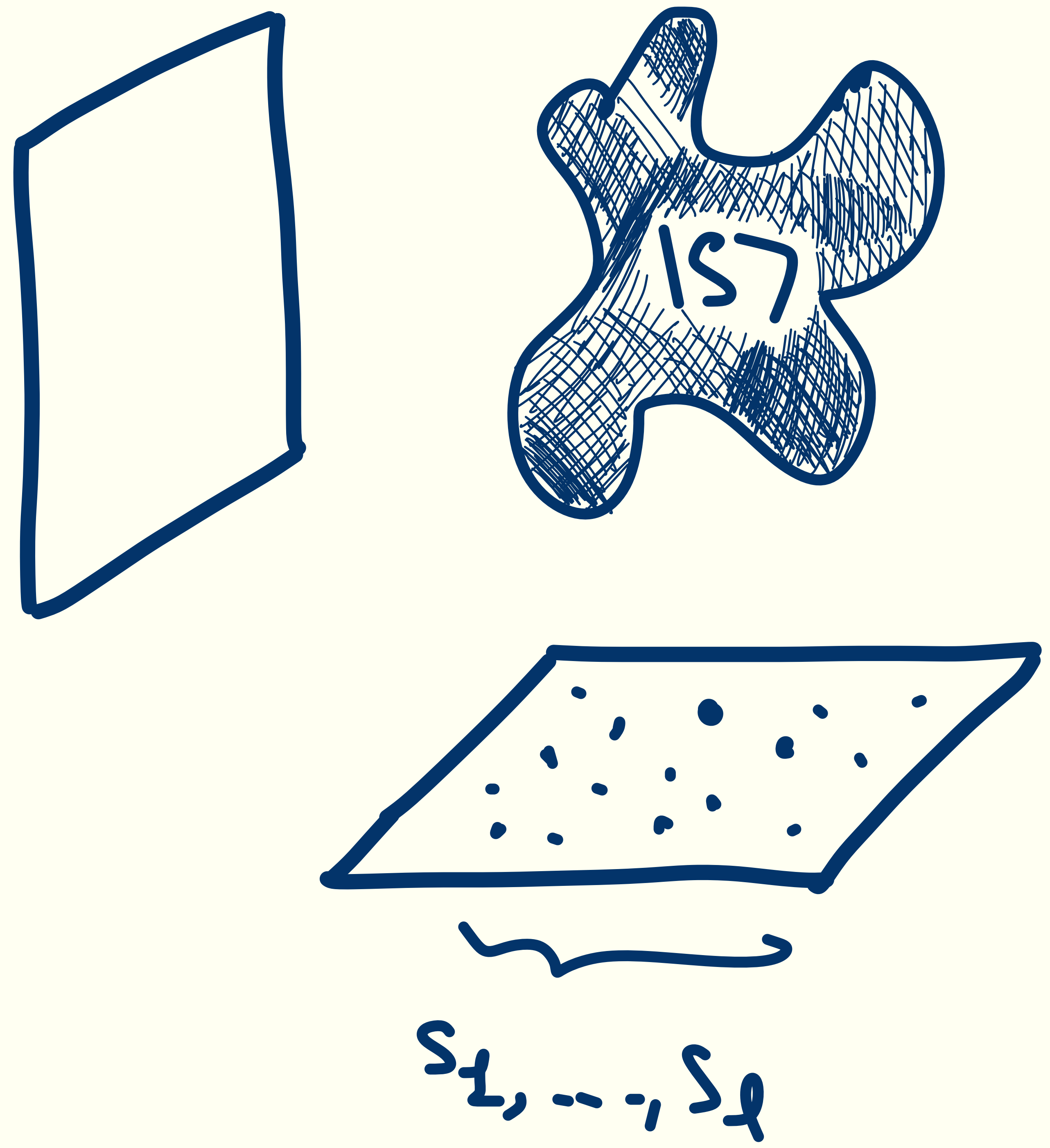

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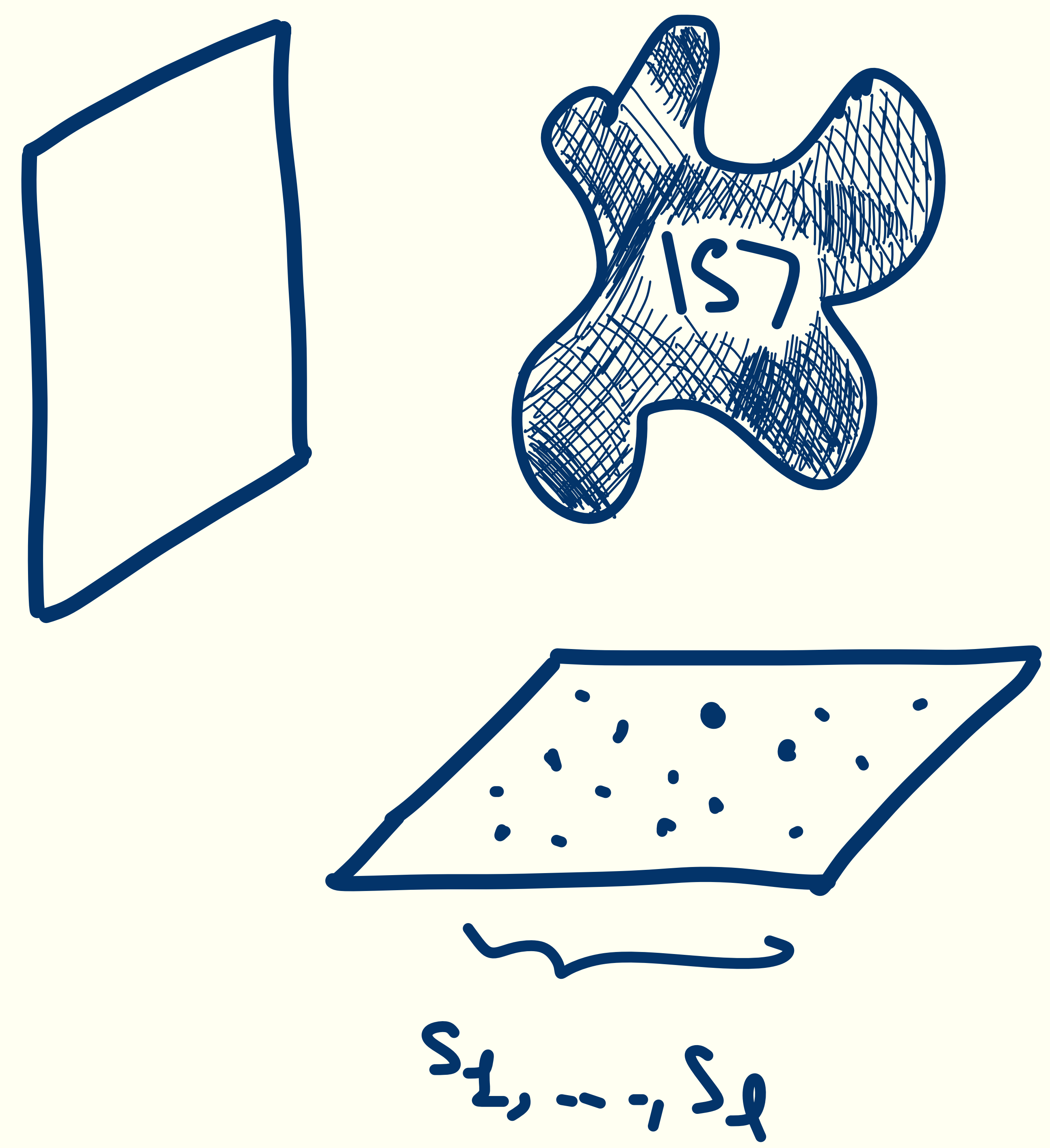
- We take U to be the heavy points of $H^{\otimes n}|S\rangle$, the Hadamard transform of $|S\rangle$.



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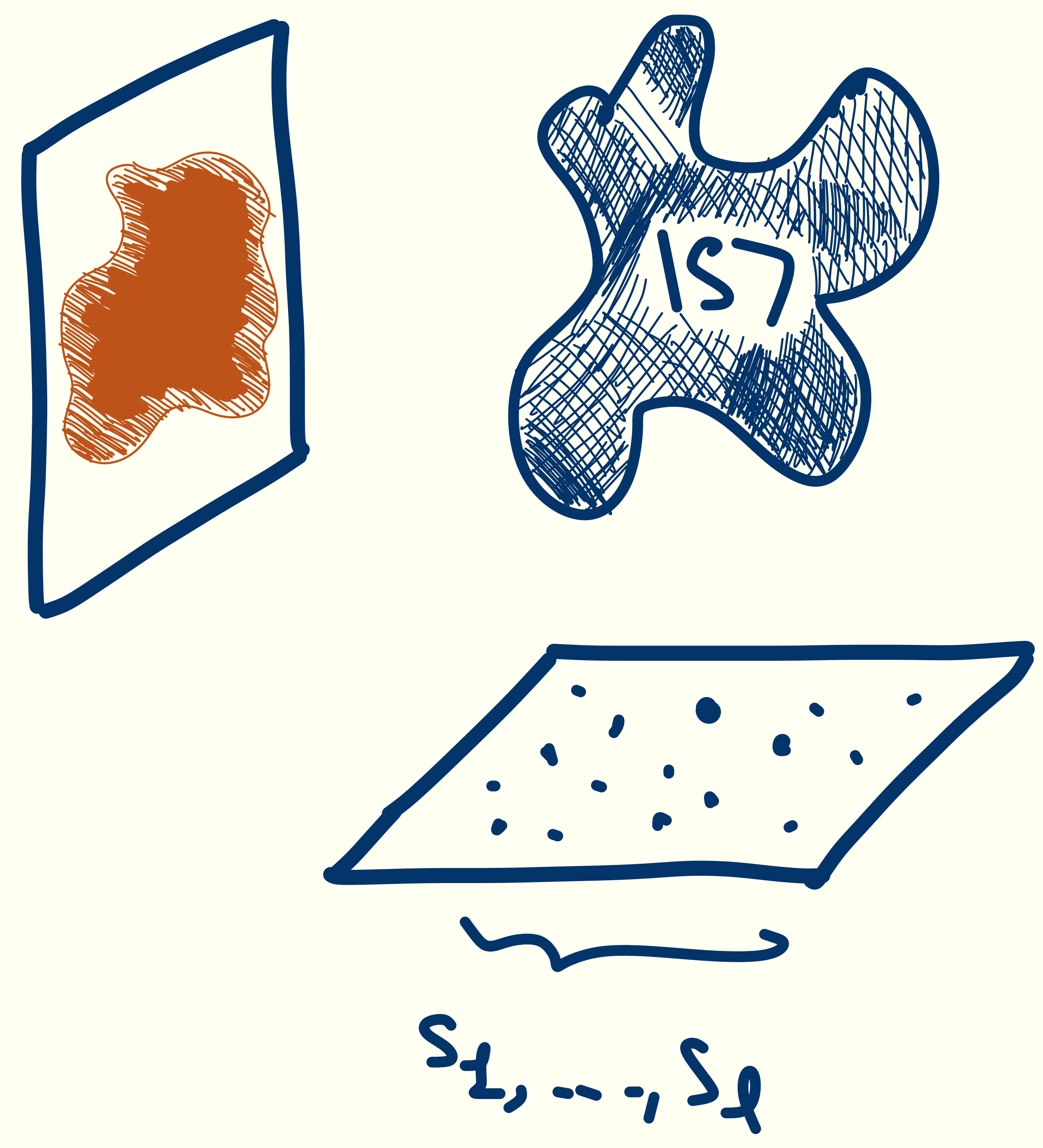

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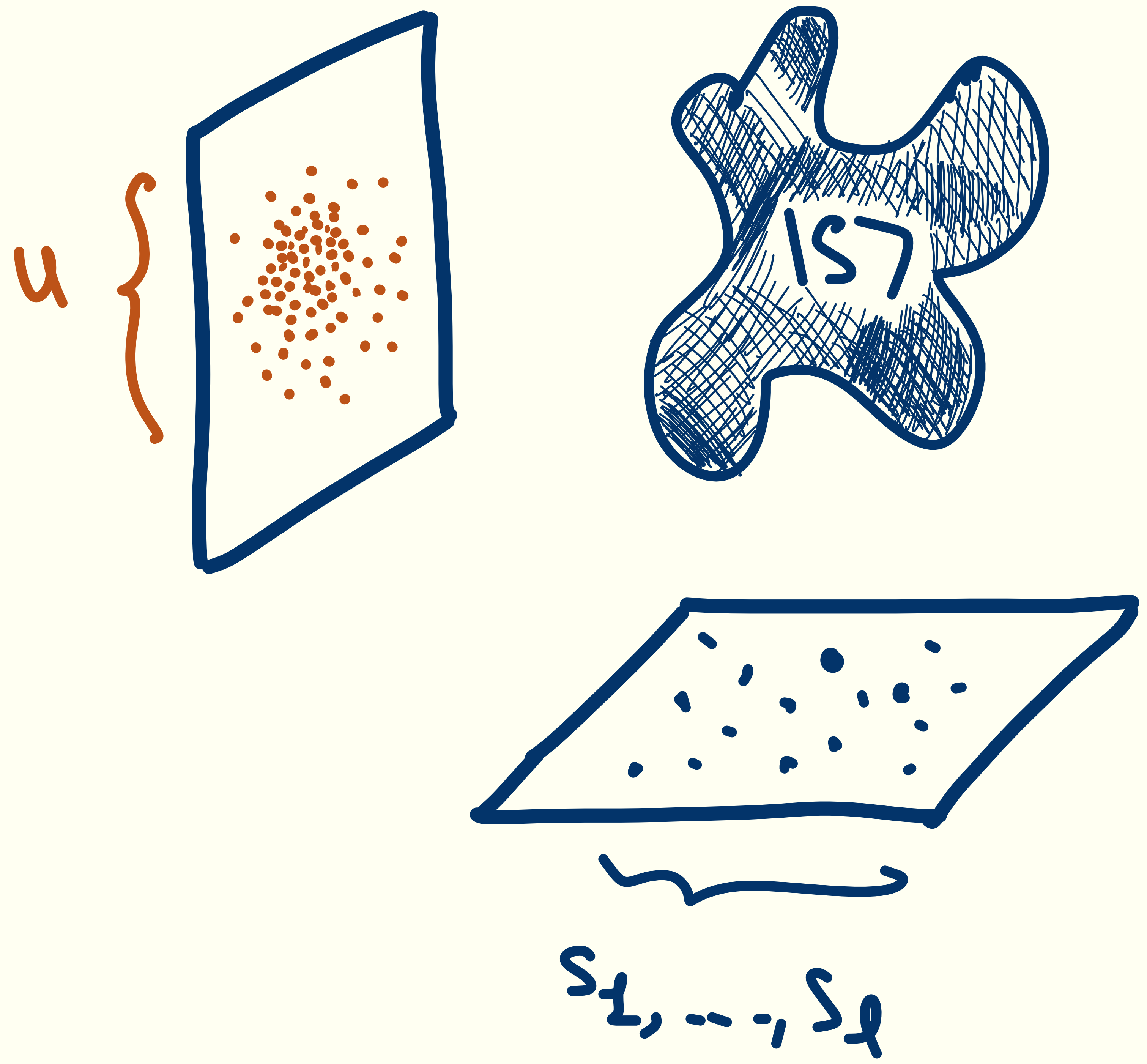

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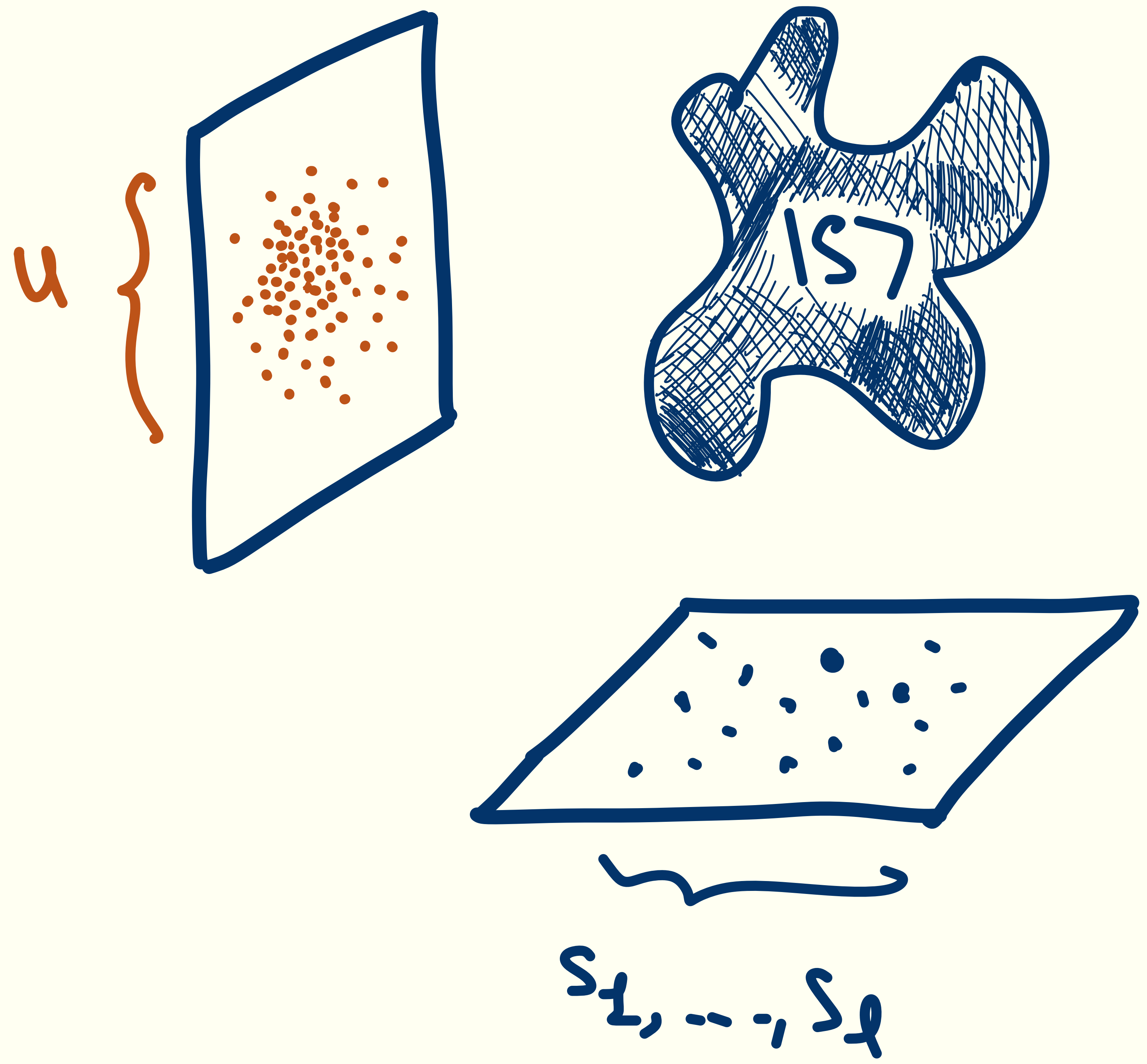
We call this distribution over oracles the Strong distribution.



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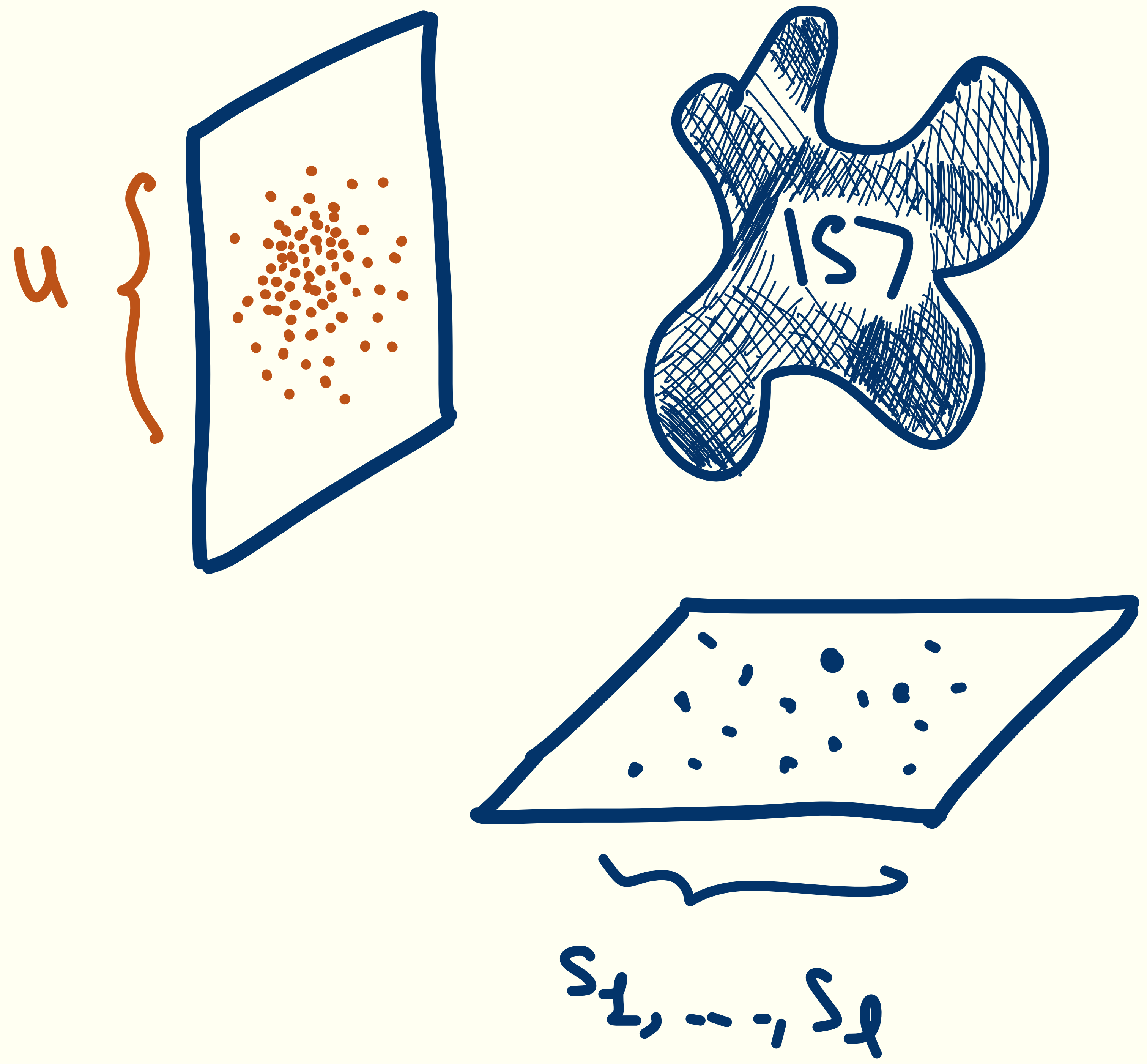
Claim (Informal): For $(S, U) \sim$ Strong, the following holds:

$$\mathbb{E}_U \left[\Pi_S \cdot H^{\otimes n} \cdot \Pi_U \cdot H^{\otimes n} \cdot \Pi_S\right] \approx \frac{1}{10}|S\rangle\langle S| + \frac{1}{2}\mathrm{id}$$



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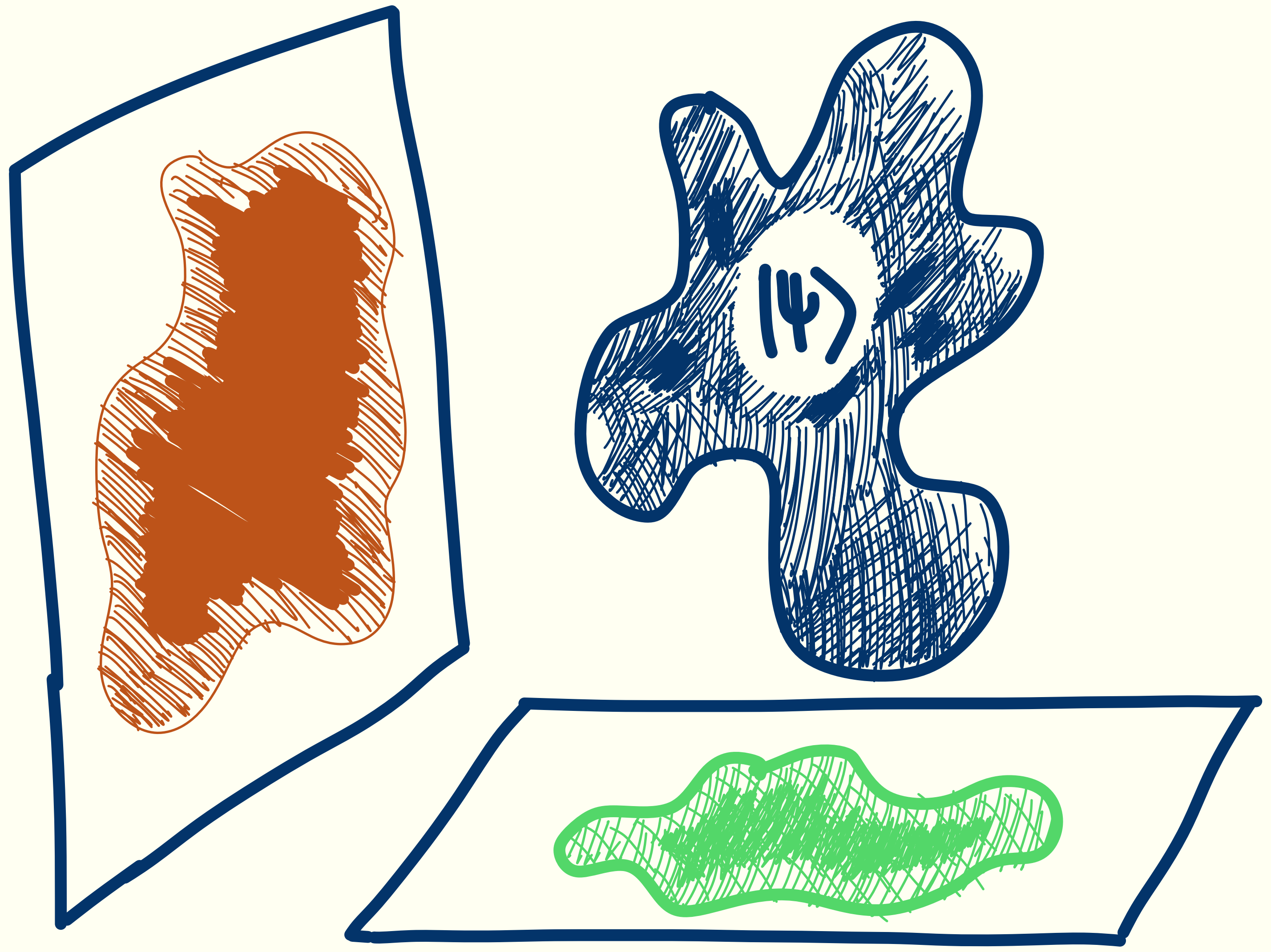

Strong yes instances

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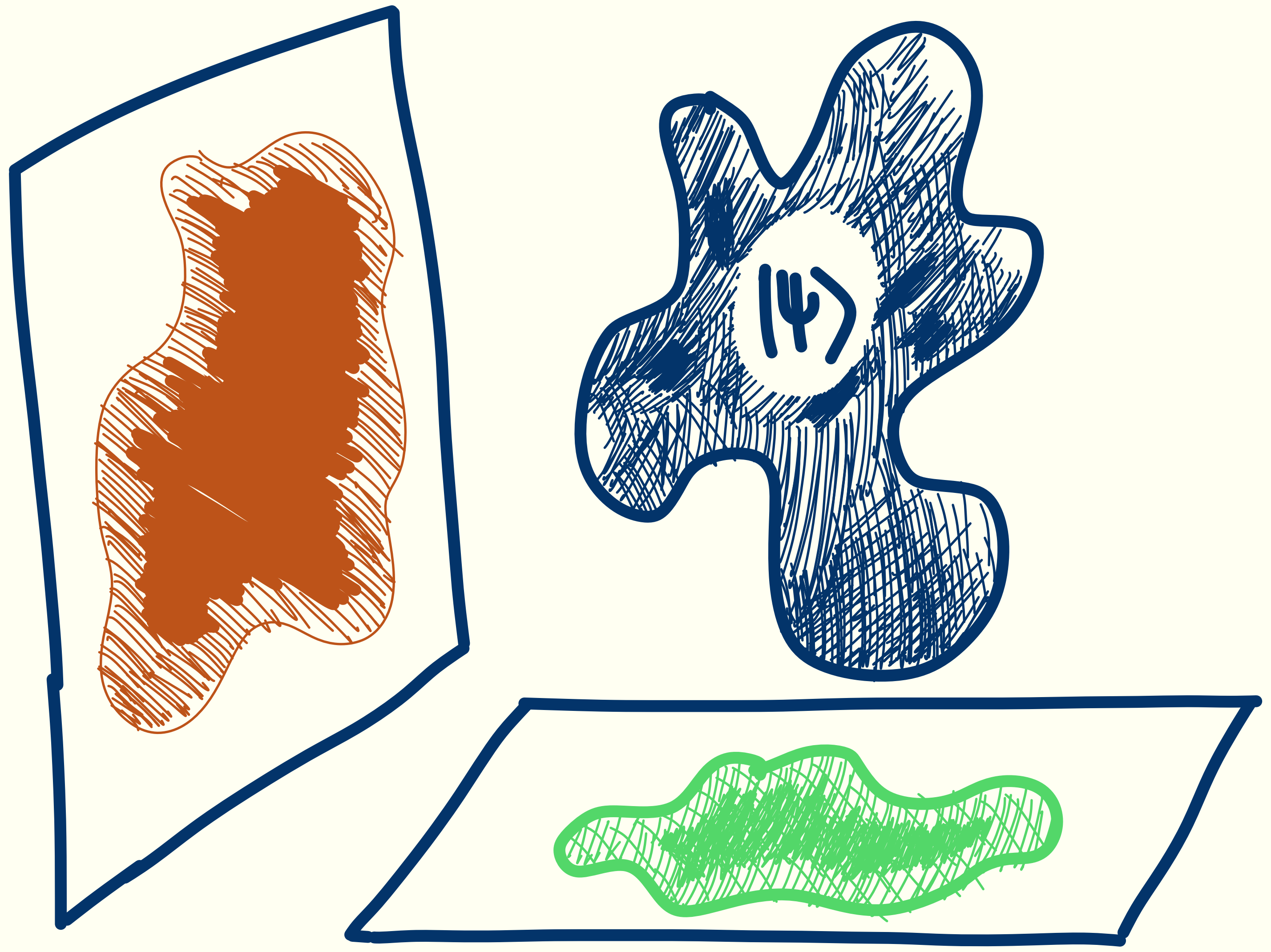
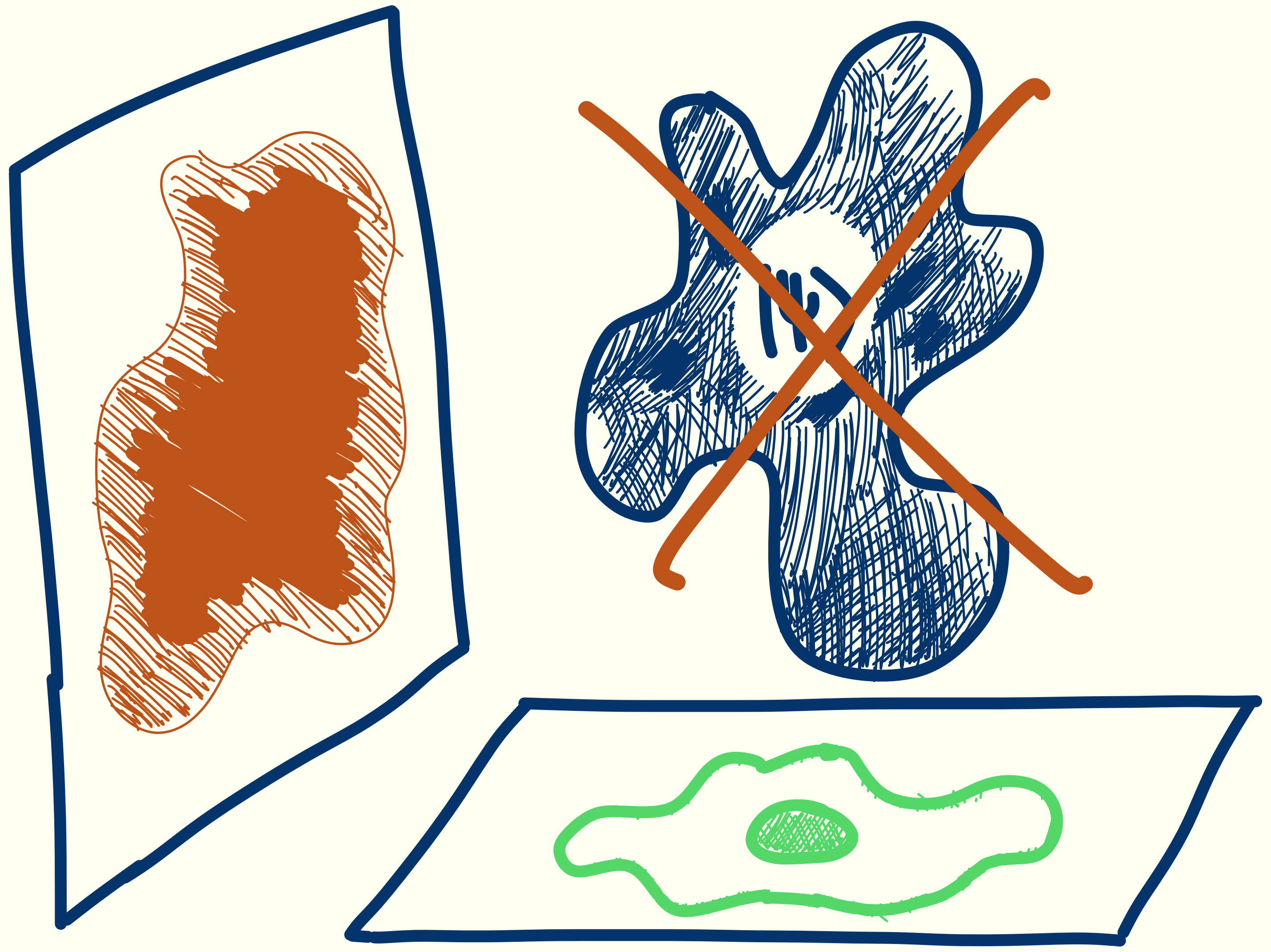
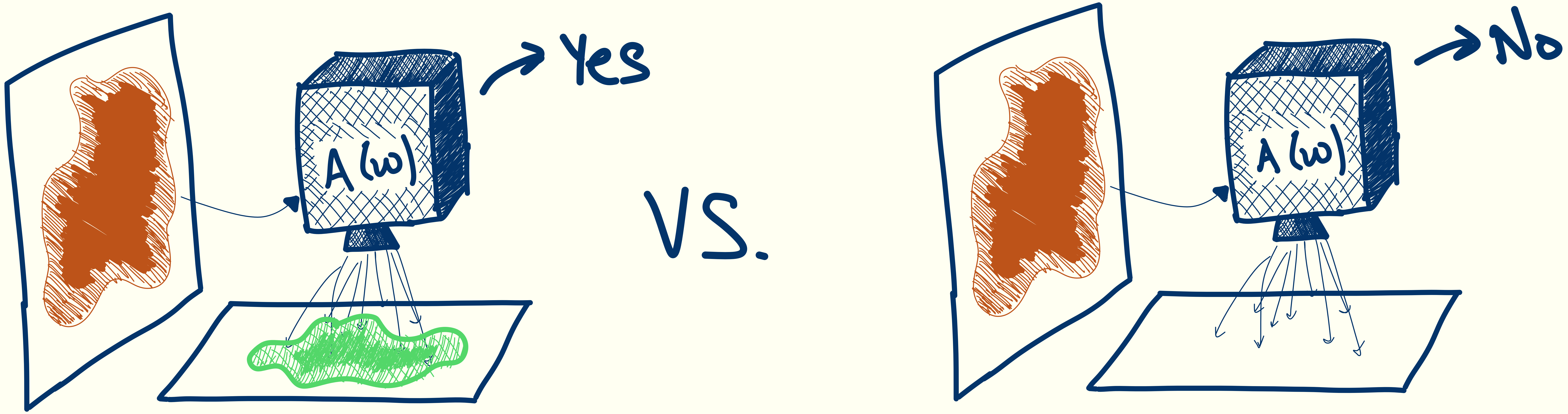
- (S, U) is a yes instance of spectral Forrelation (i.e., $\geq 59/100$ spectrally Forrelated).
- For all $\Delta \subset S$ with $|\Delta| \leq \ell/100$, (Δ, U) is a no instance of spectral Forrelation (i.e., $\leq 57/100$ spectrally Forrelated).





Strong yes instances can be sampled from

Any quantum query algorithm that distinguishes between (S, U) and $(\varnothing, U)$ must query a point in S pretty often ($\geq 1/3t$ chance per query), since otherwise the action of the oracles is identical.



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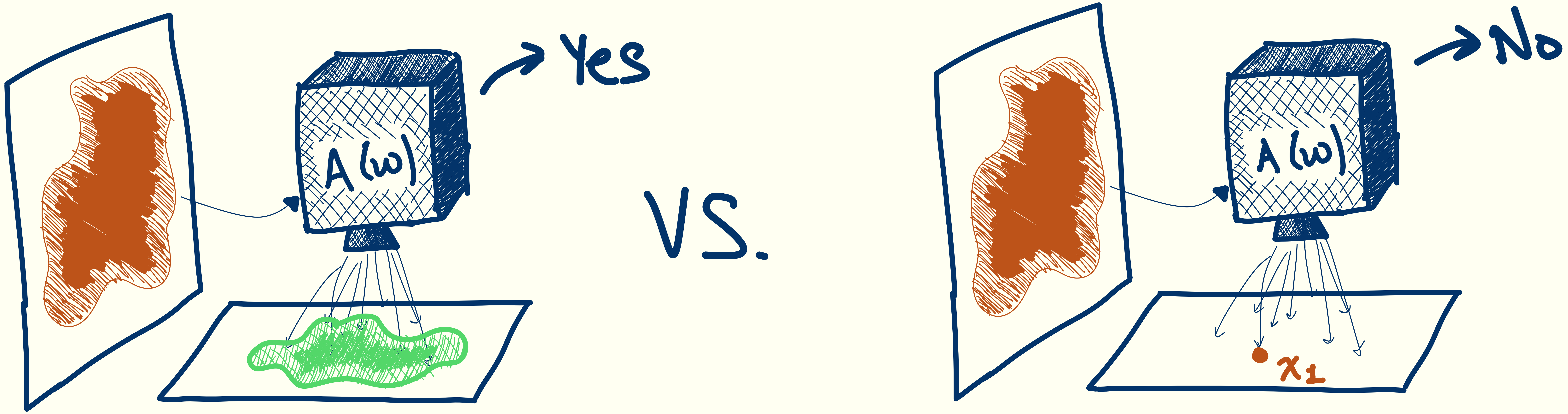

Therefore, measuring a random query of the algorithm will yield a point in S with good probability, x_1.

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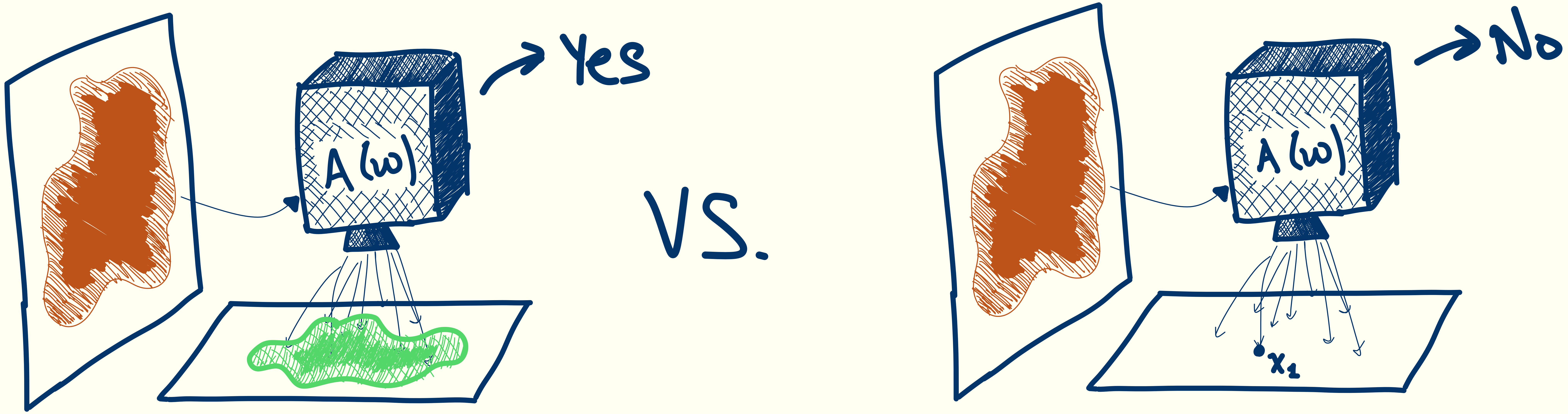

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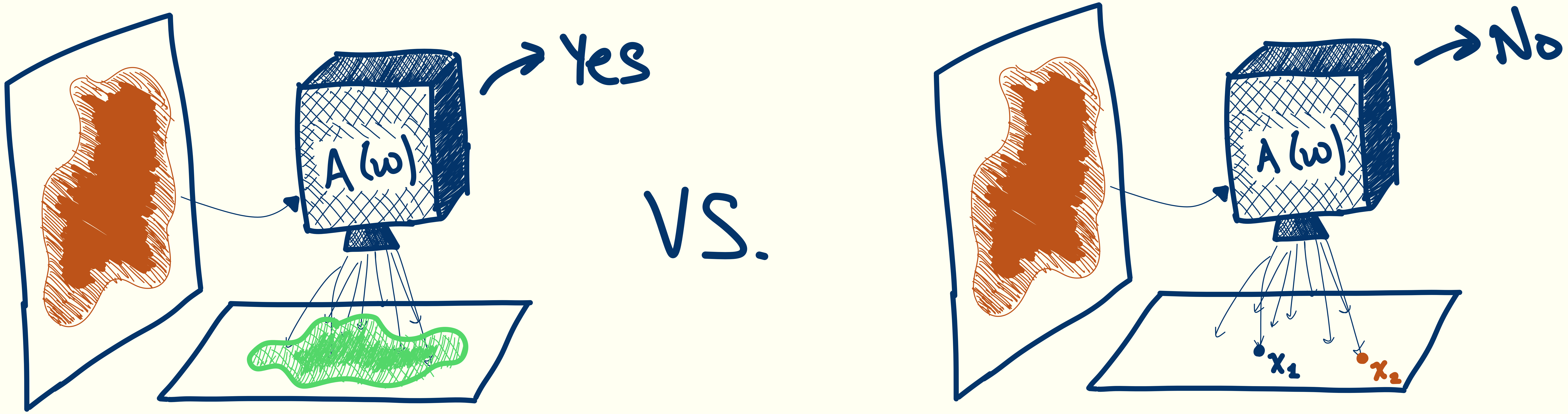
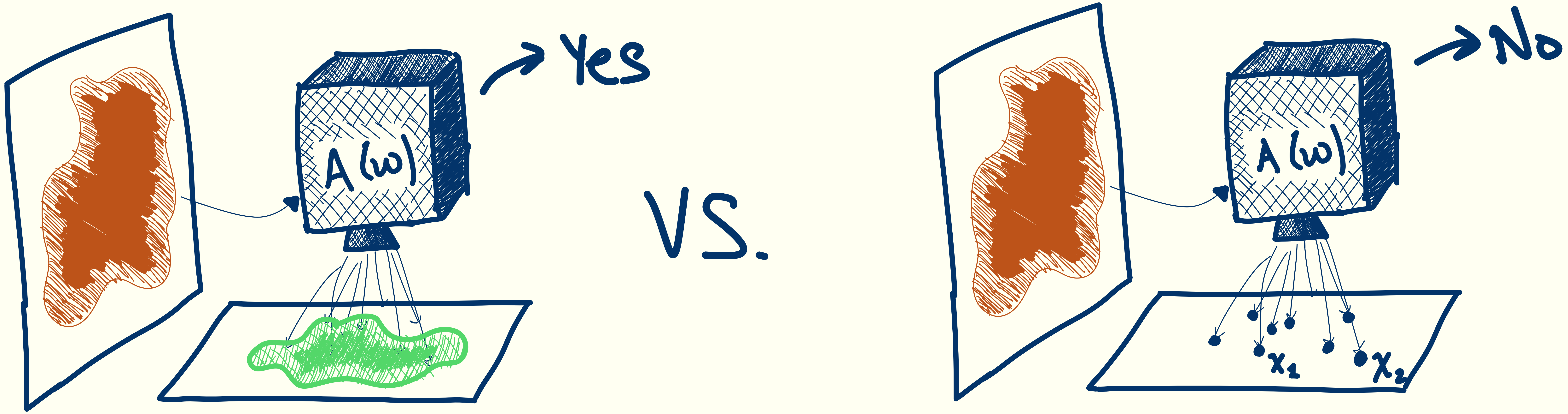


Therefore, measuring a random query of the algorithm will yield a point in S with good probability, x_2.

Strong yes instances can be sampled from

Because of the strong yes property, we can keep going until $\ell/100$ points have been sampled! This is the part that uses the fact that the witness is classical.



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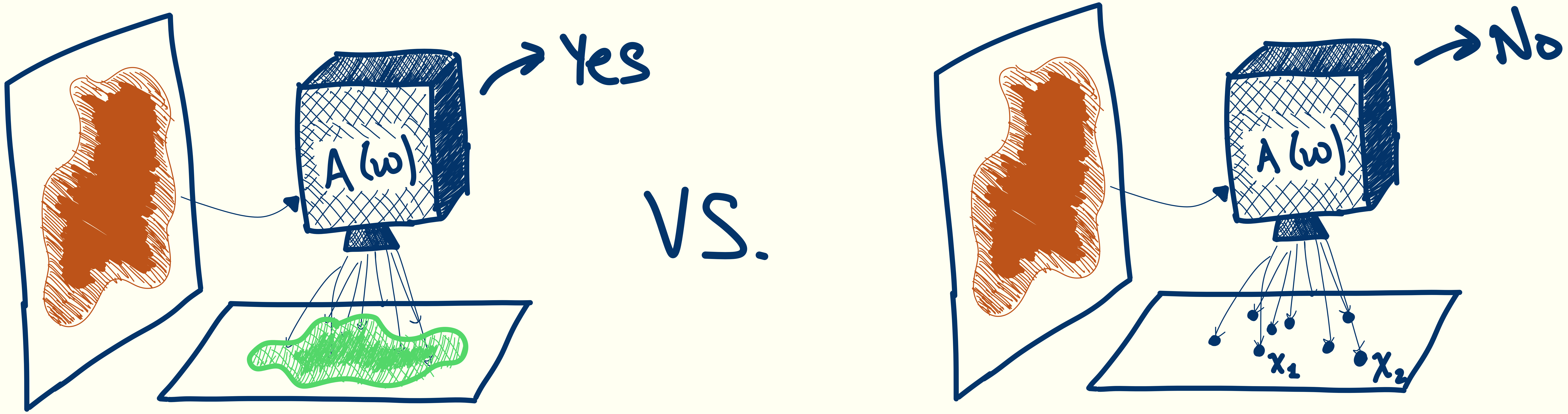

Given a QCMA algorithm, we can guess the classical witness and be correct with probability 2^{-q}.

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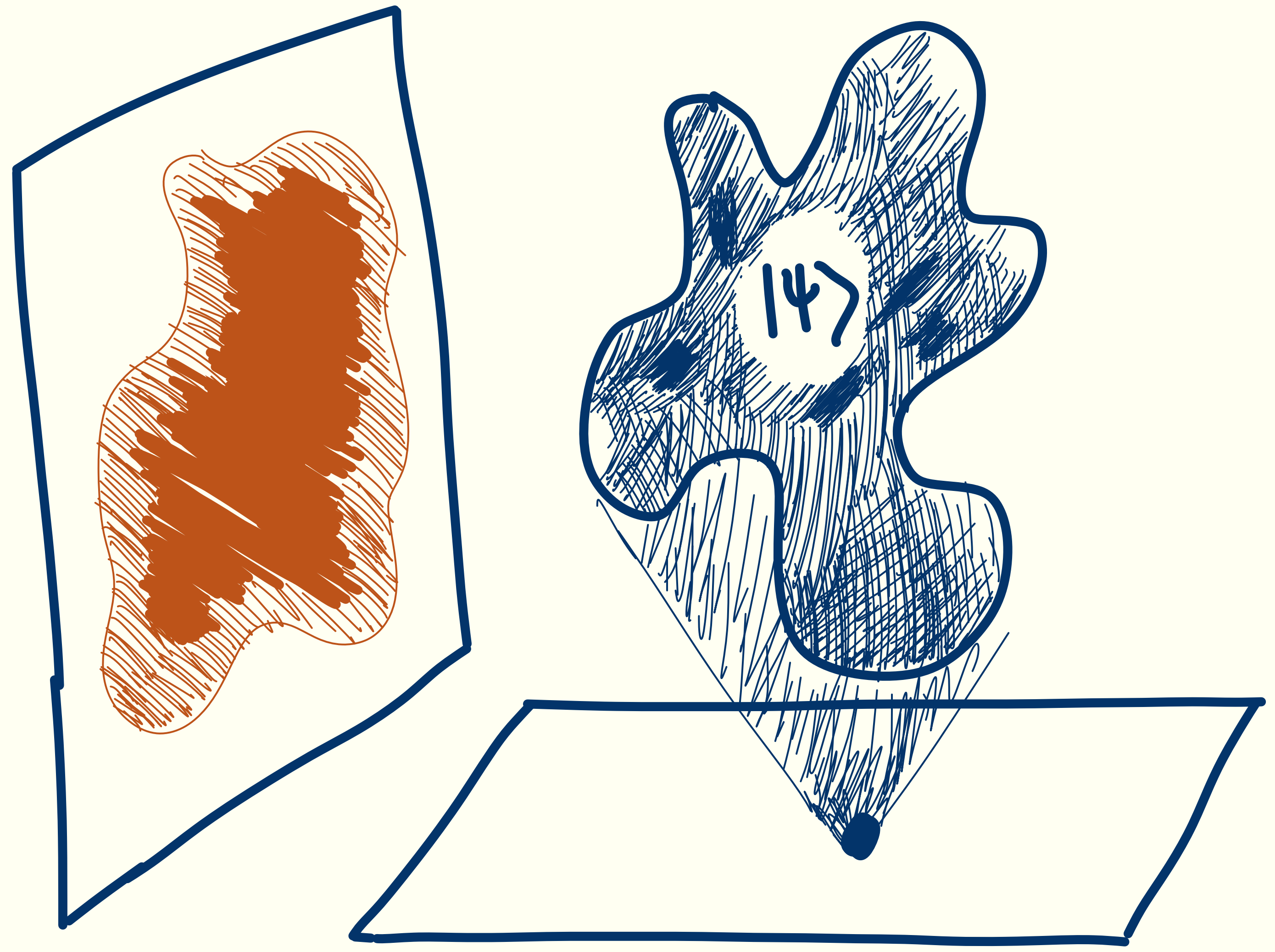

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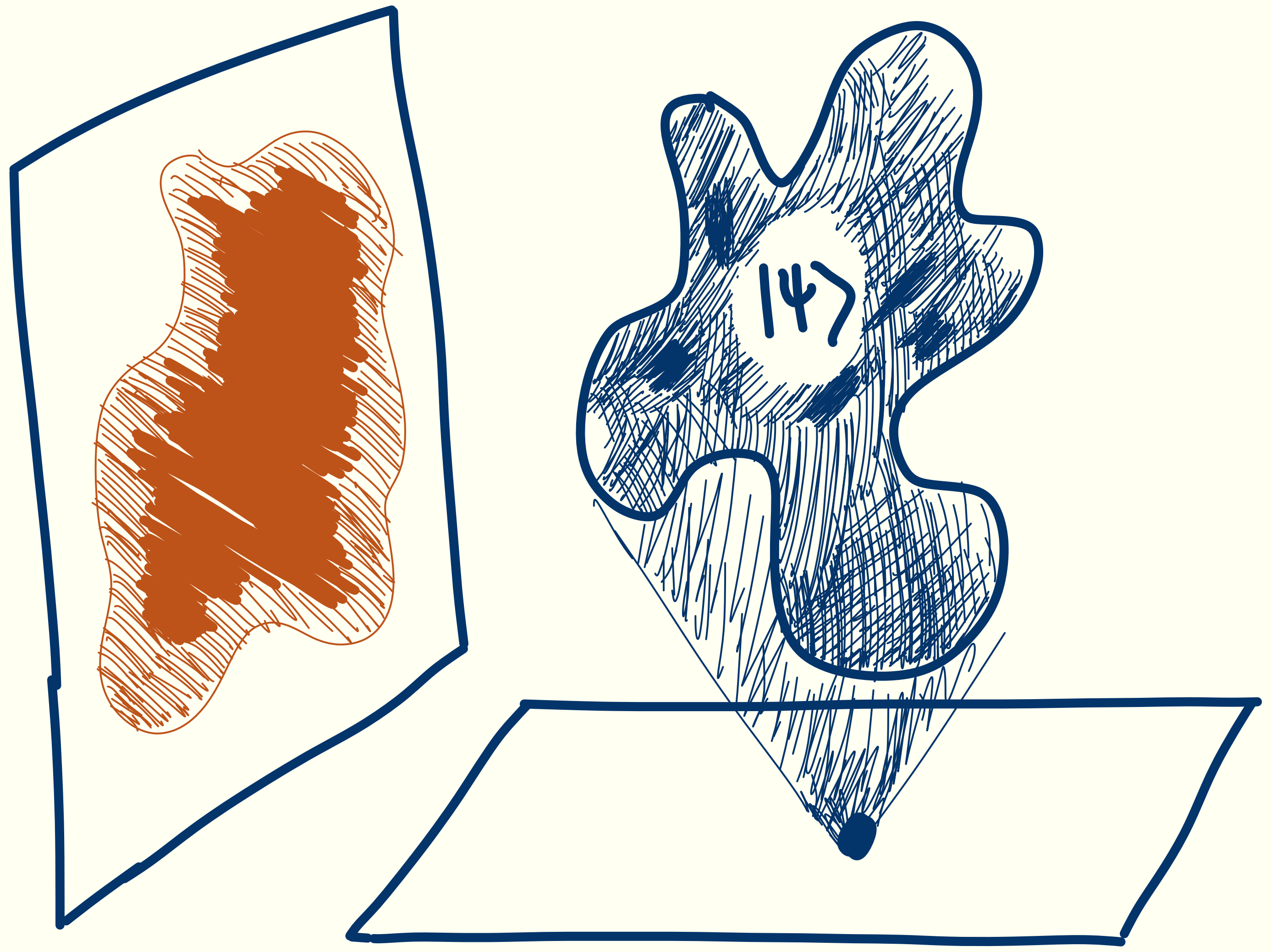
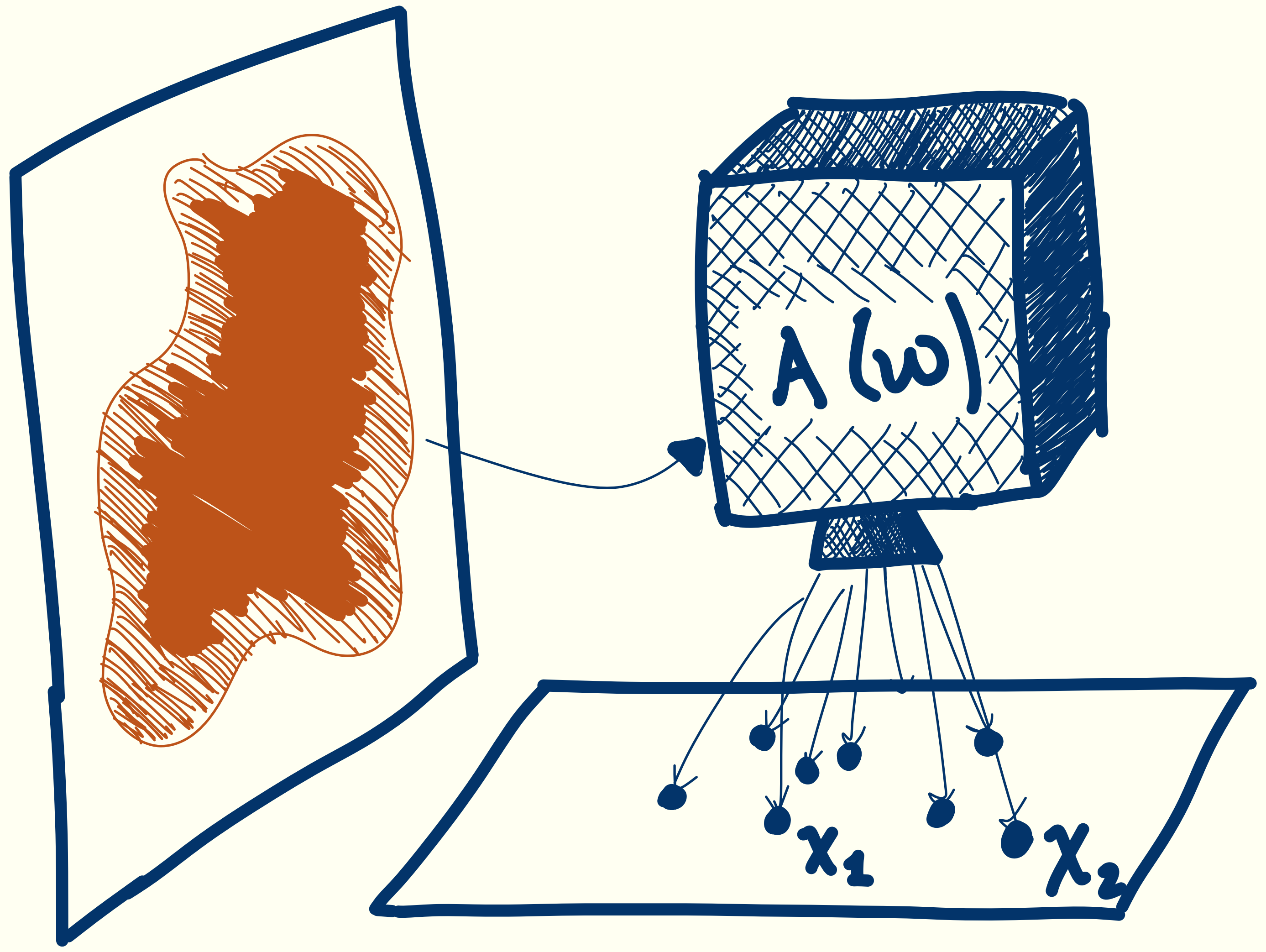


A classical witness that help you verify spectral Forrelation can be re-used, must actually specify $\ell/10$ points from S, somehow!



Main theorems

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Theorem 1: For all $v > 0$, and all quantum query algorithms making $T = T(n)$ queries to a set membership oracle for U, the probability, over Strong, that the algorithm outputs v distinct points from S is at most

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Theorem 2: If there exists a QCMA algorithm, making $t = t(n)$ queries to (S, U) and taking a witness of length $q = q(n)$, then for all $0 < v < \ell/100$, there is a query algorithm making vt queries to U that outputs v distinct points from S with probability

$$\geq 2^{-q}\left(\frac{1}{36t^2}\right)^v$$

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→ This removal of structure allowed us to understand queries to the Fourier transform of an oracle way better than we could before!
- **Much more work is needed!**
→ Understanding oracles with structure seems to require an understanding that structure, seem to be annoying to deal with using general methods.
→ To understand other oracles (expander mixing problem, Yamakawa-Zhandry, etc.), we will need more specific tools, or a big leap in understanding of quantum algorithms.

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  → Our ideas lie in the intersection of ideas used for quantum money (subset states ↔ subspace states, Fourier transform of S ↔ Fourier transform for group actions).
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- **Is there a connection to the Aaronson-Ambainis conjecture?**
  → Both Liu-Mutreja-Yuen'24 and Zhandry'24 showed that there is a connection between QCMA versus QMA and pseudorandomness against quantum algorithms.
  → Our proof didn't say anything about this, but could you use our techniques?

Thanks for listening!