Quantum query complexity, with advice

John Bostanci


HOMER HERODOTUS SOPHOCLES PLATO ARISTOTLE DEMOSTHENES CICERO VERGIL


I love you!


HOMER HERODOTUS SOPHOCLES PLATO ARISTOTLE DEMOSTHENES CICERO VERGIL

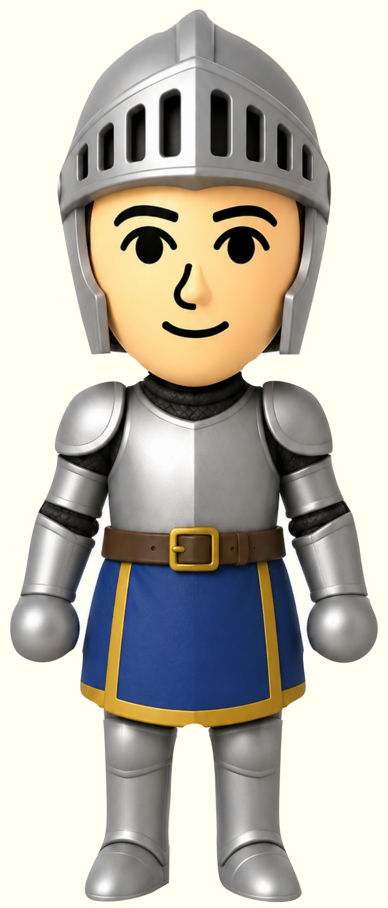

HOMER HERODOTUS SOPHOCLES PLATO ARISTOTLE DEMOSTHENES CICERO VERGIL


Prove it!


(mom ⟹ love)
mom
∴ love

Proofs in computer science

Efficient Classical Algorithm



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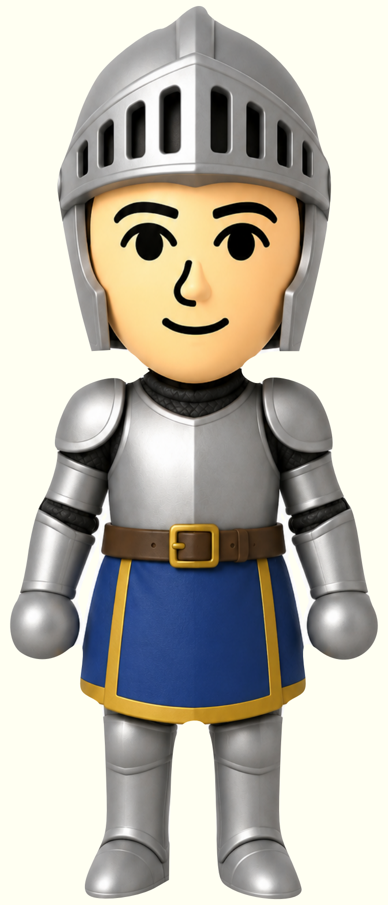

Unbounded prover

Classical proof

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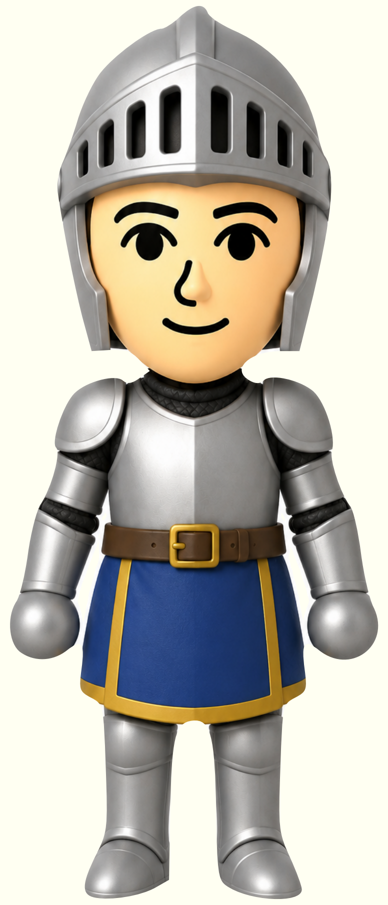

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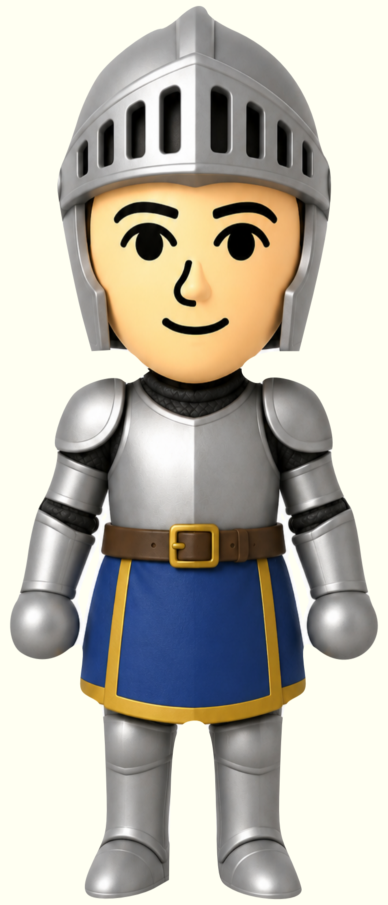

P vs NP

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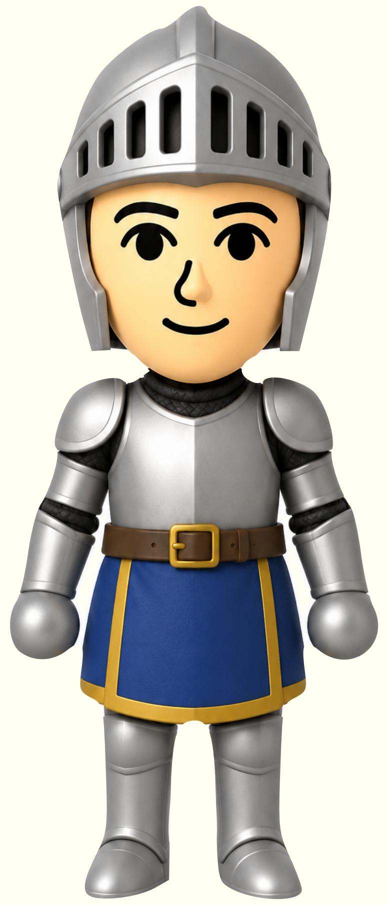

Proofs in quantum computer science

Efficient Classical Algorithm

P vs BQP

Efficient Quantum Algorithm

BQP vs NP

Unbounded prover

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Proofs in quantum computer science

Efficient Classical Algorithm

P vs BQP

$N = pq$

Efficient Quantum Algorithm

p, q

BQP vs NP

Unbounded prover

Efficient Classical Algorithm

Proofs in quantum computer science

Efficient Quantum Algorithm

vs

Unbounded prover

Classical proof

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Classical proof

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vs

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QCMA vs QMA

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Quantum proof

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→ If there were ways to efficiently verify these with a classical proof, quantum systems have a lower “complexity” than we thought!

→ If not, then for all of these problems, you can only check them with quantum proofs!

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- Quantum algorithms are really good at detecting global structure, so it's even hard to rule out BQP algorithms for some candidate oracle separations!

A hard problem for QCMA

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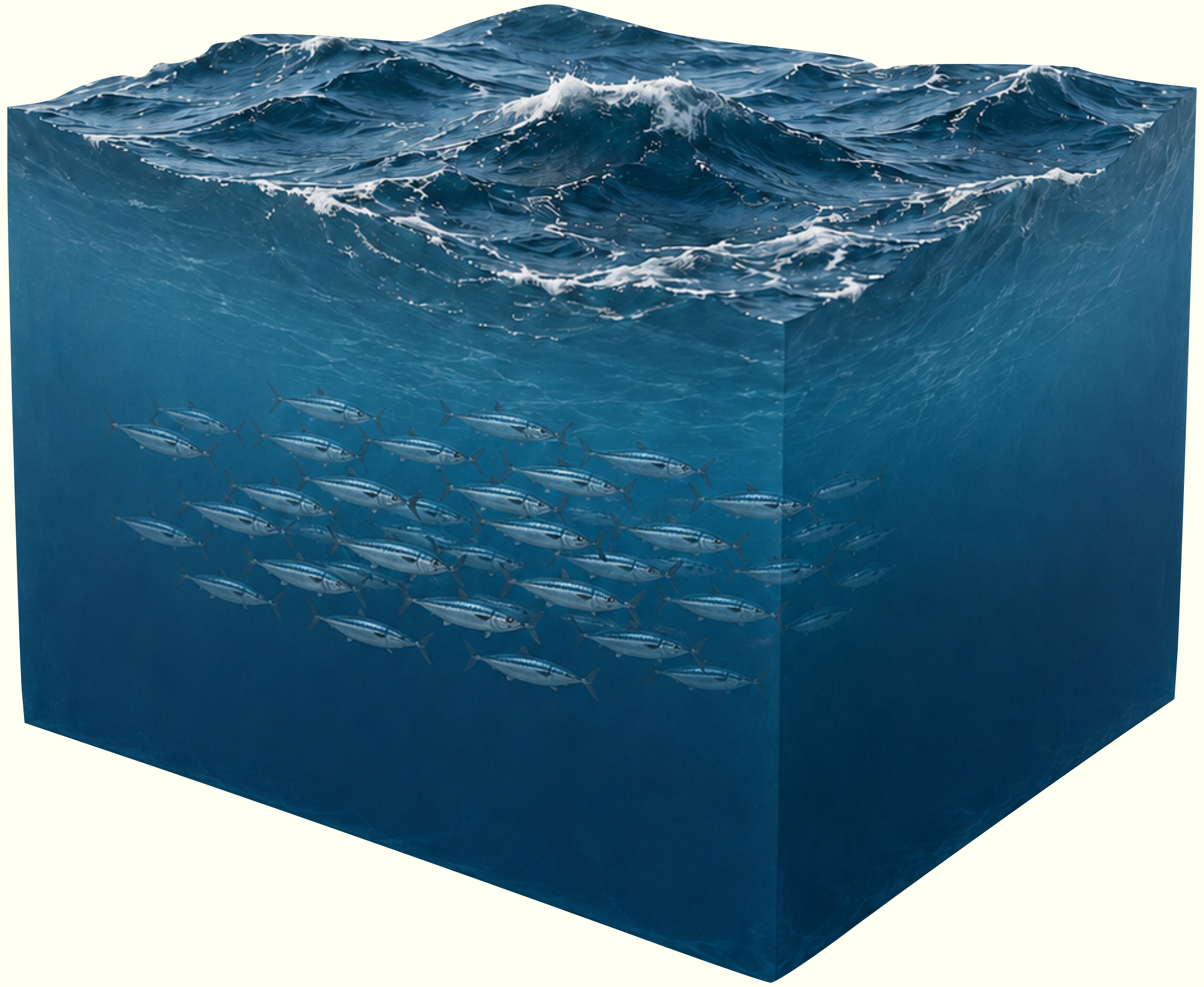
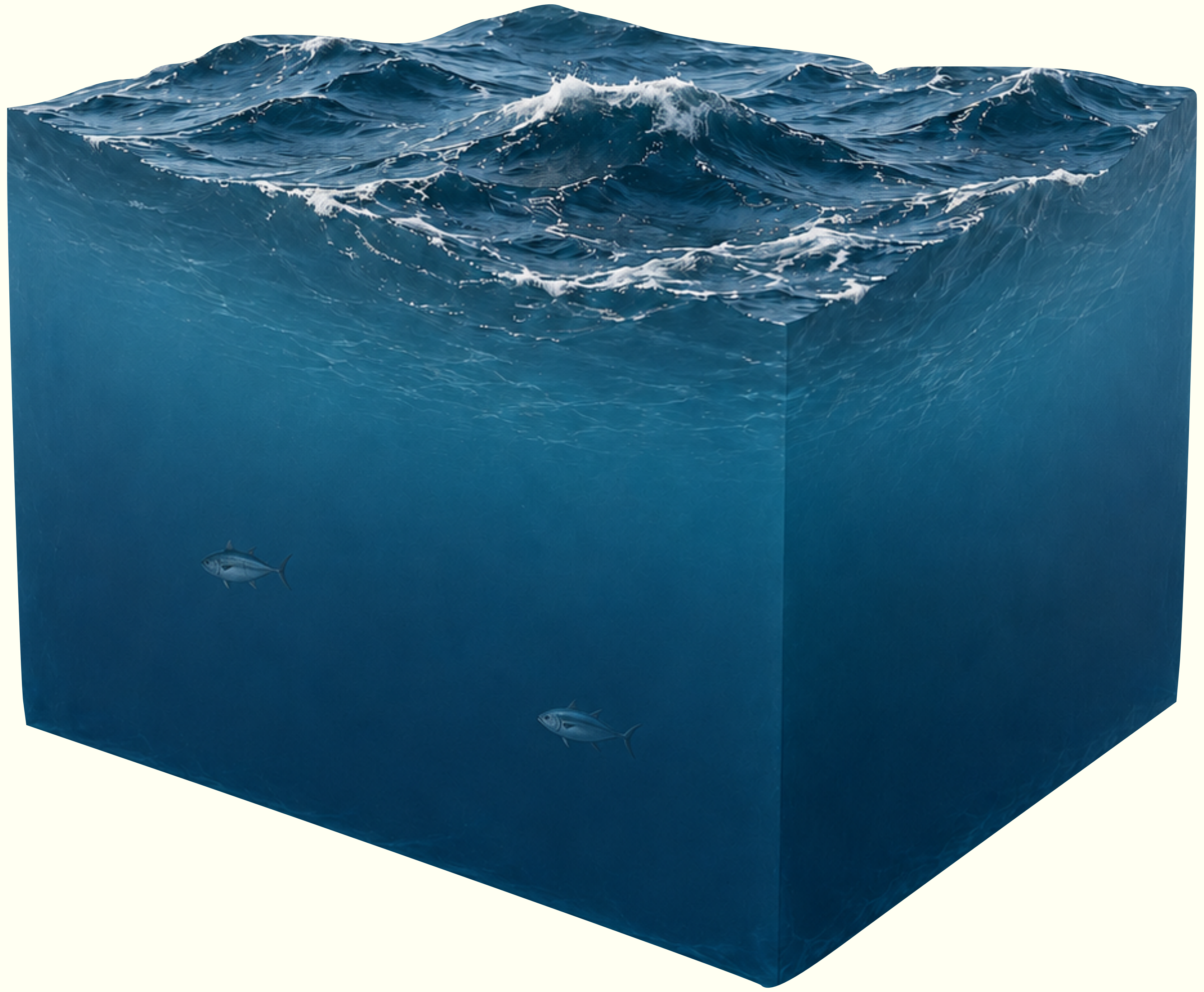


VS



A new approach for ruling out QCMA

In slightly more detail, we are considering the following problem:

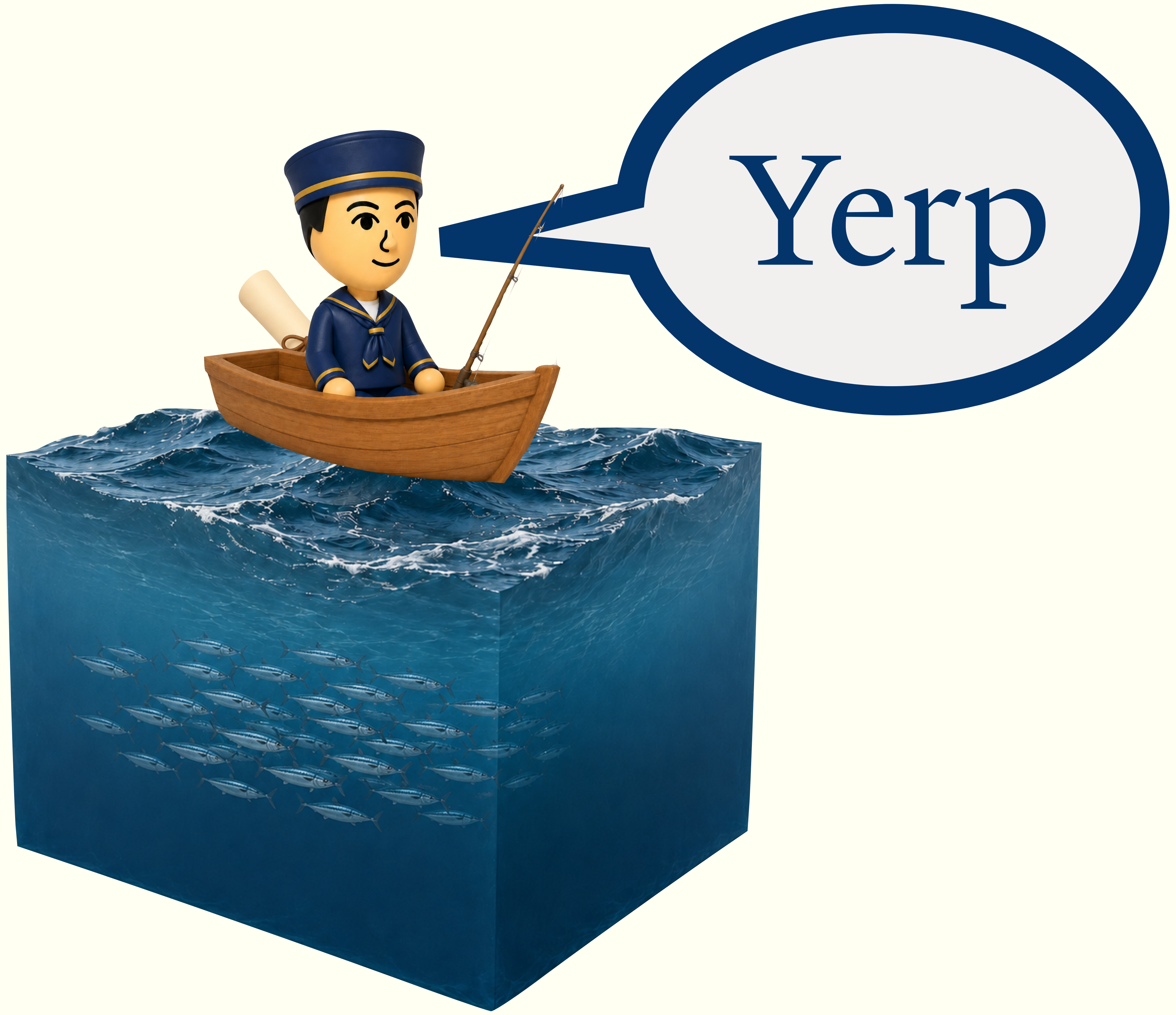
Input: Oracle access to a set S (where fish are), and size ℓ (huge!)

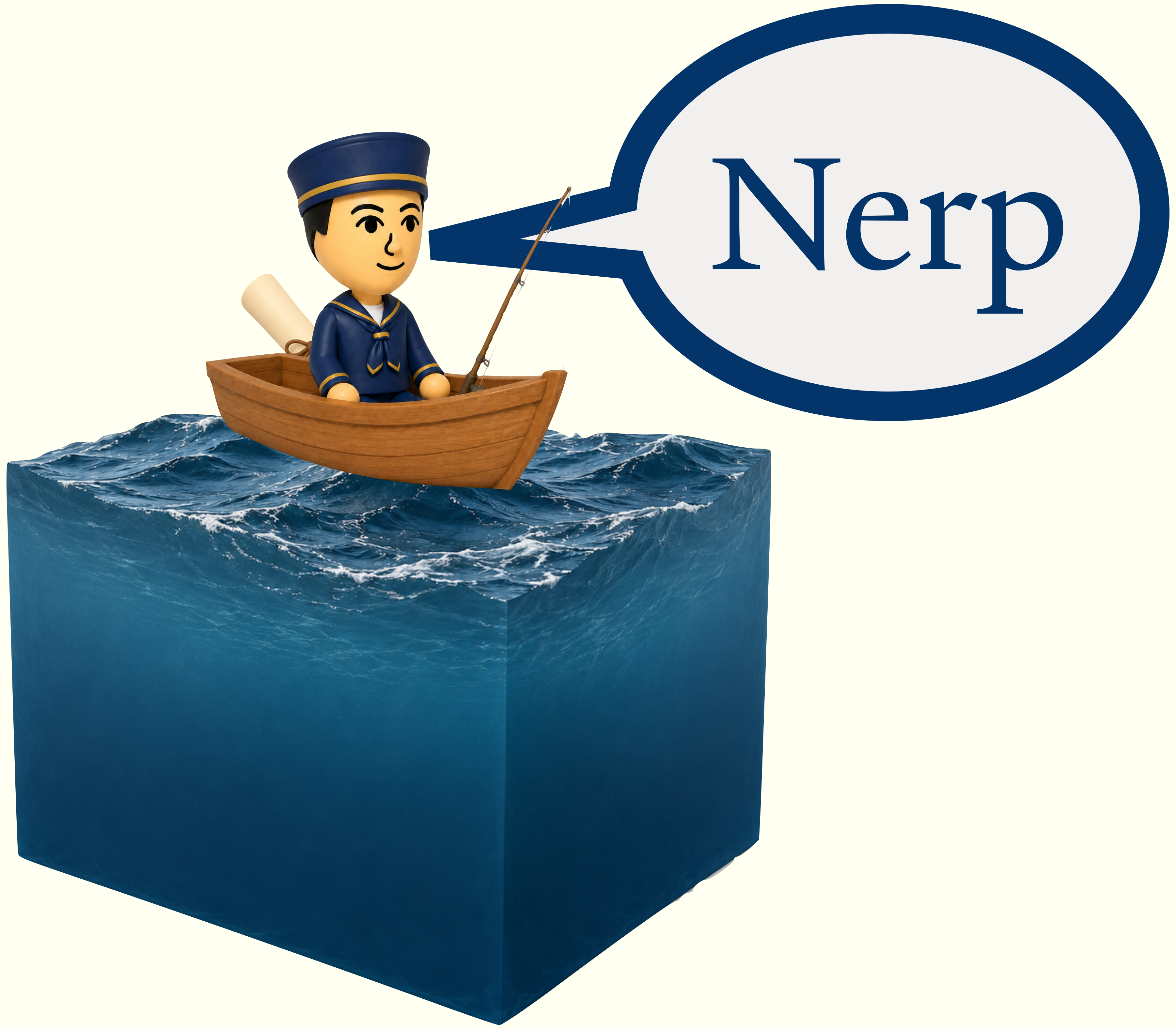
Output: Is $|S| \geq \ell$ or $|S| \leq \ell/2$, promised one of the two is the case.

VS

A new approach for ruling out QCMA

Assume for the sake of contradiction that the sailor (with classical witness) would successfully solve this problem, then he definitely distinguishes between many fish and no fish…





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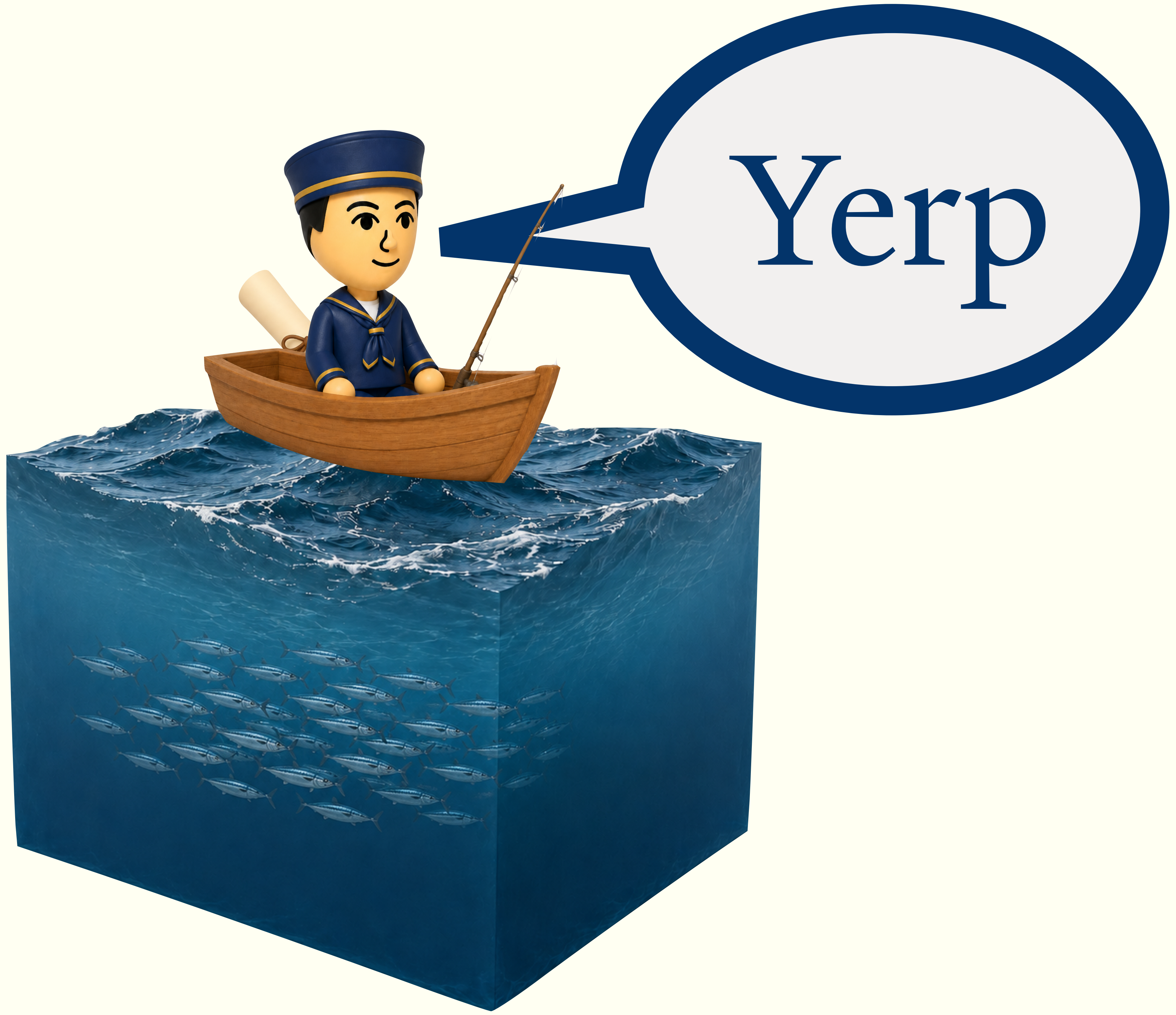

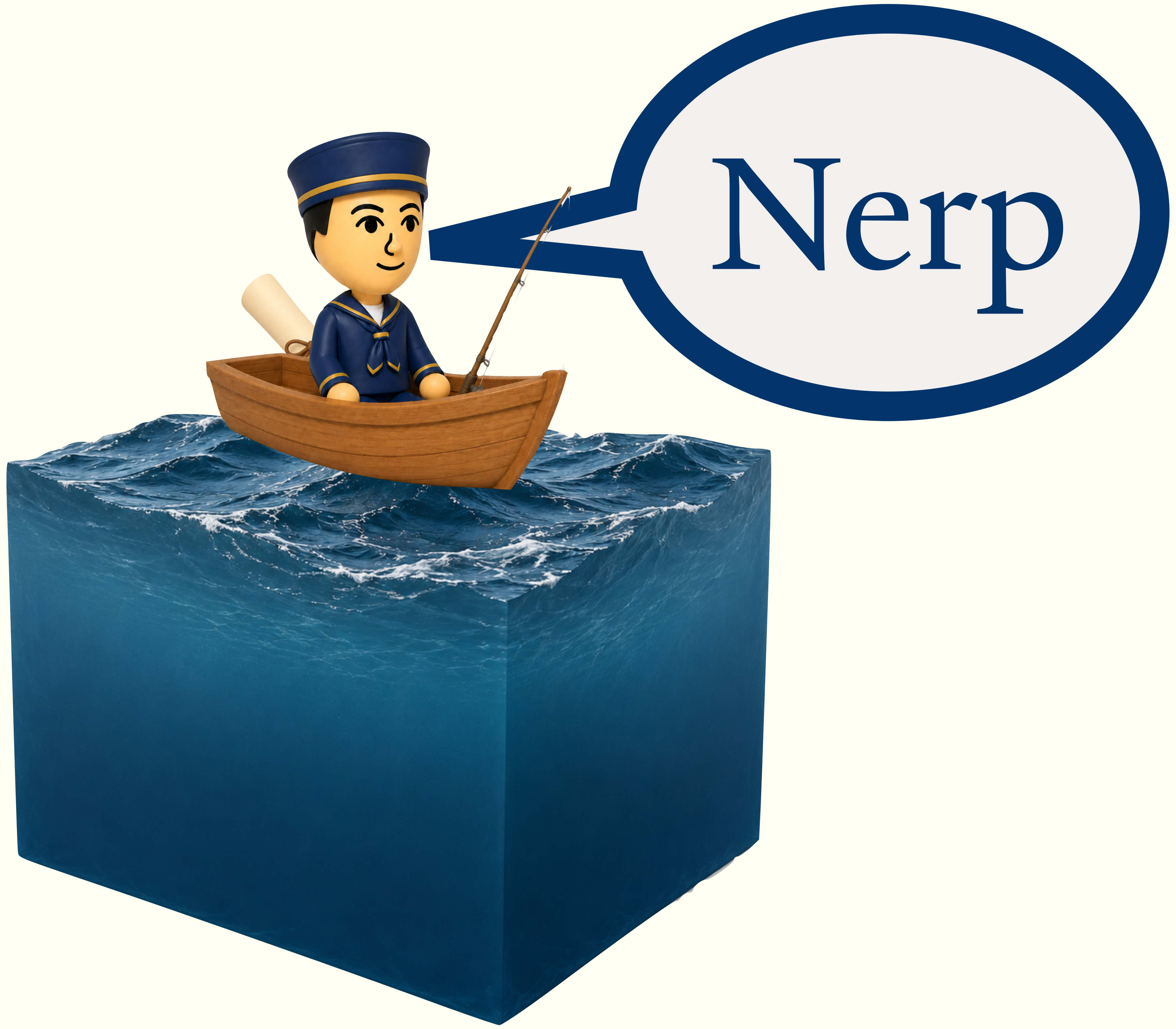

If the sailor never catches a fish, they would not distinguish between the two cases!

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Imagine ``simulating'' the problem in a pool: Let the sailor move around as if they were in the ocean, but any time they fish they won't catch anything.

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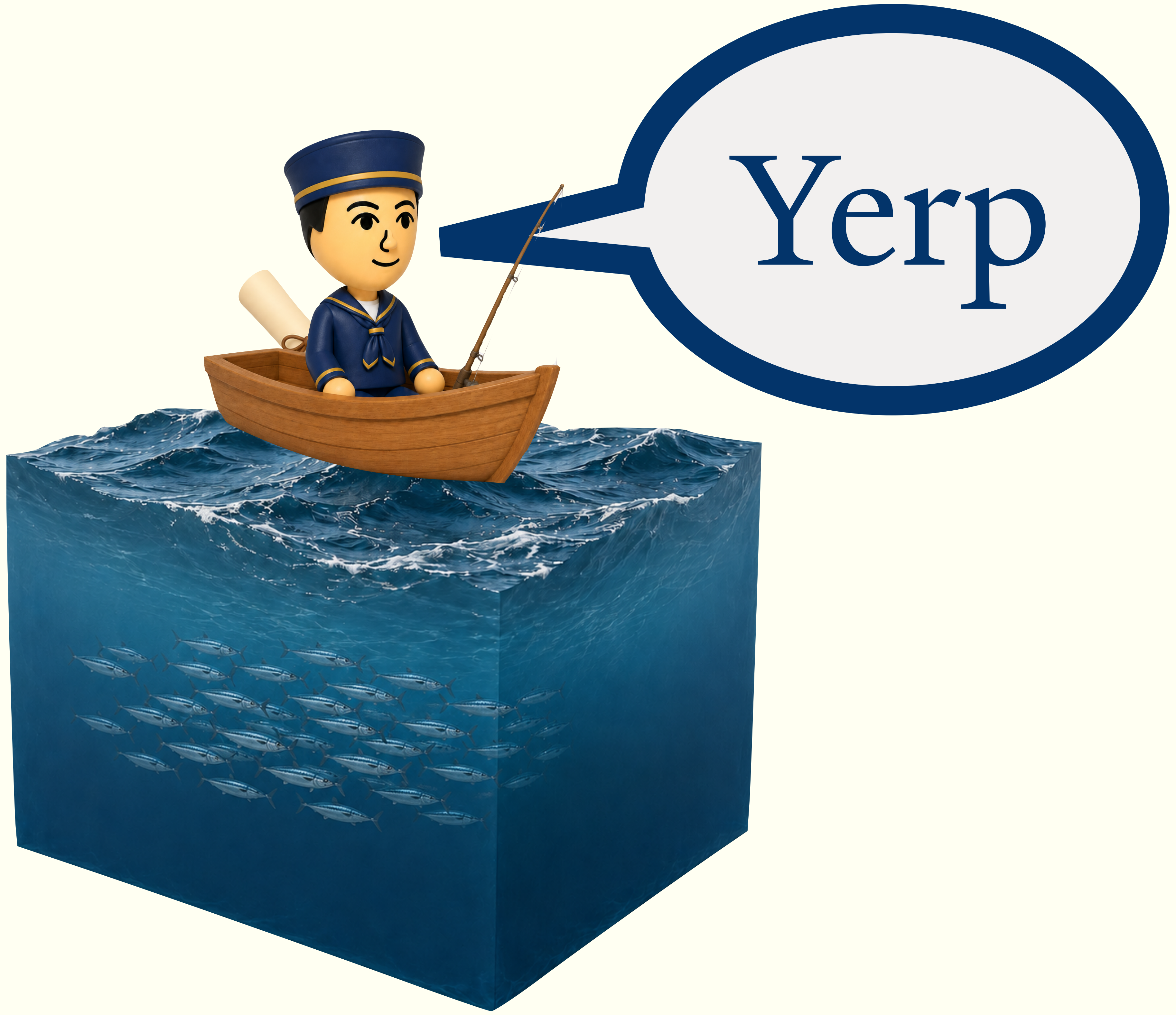
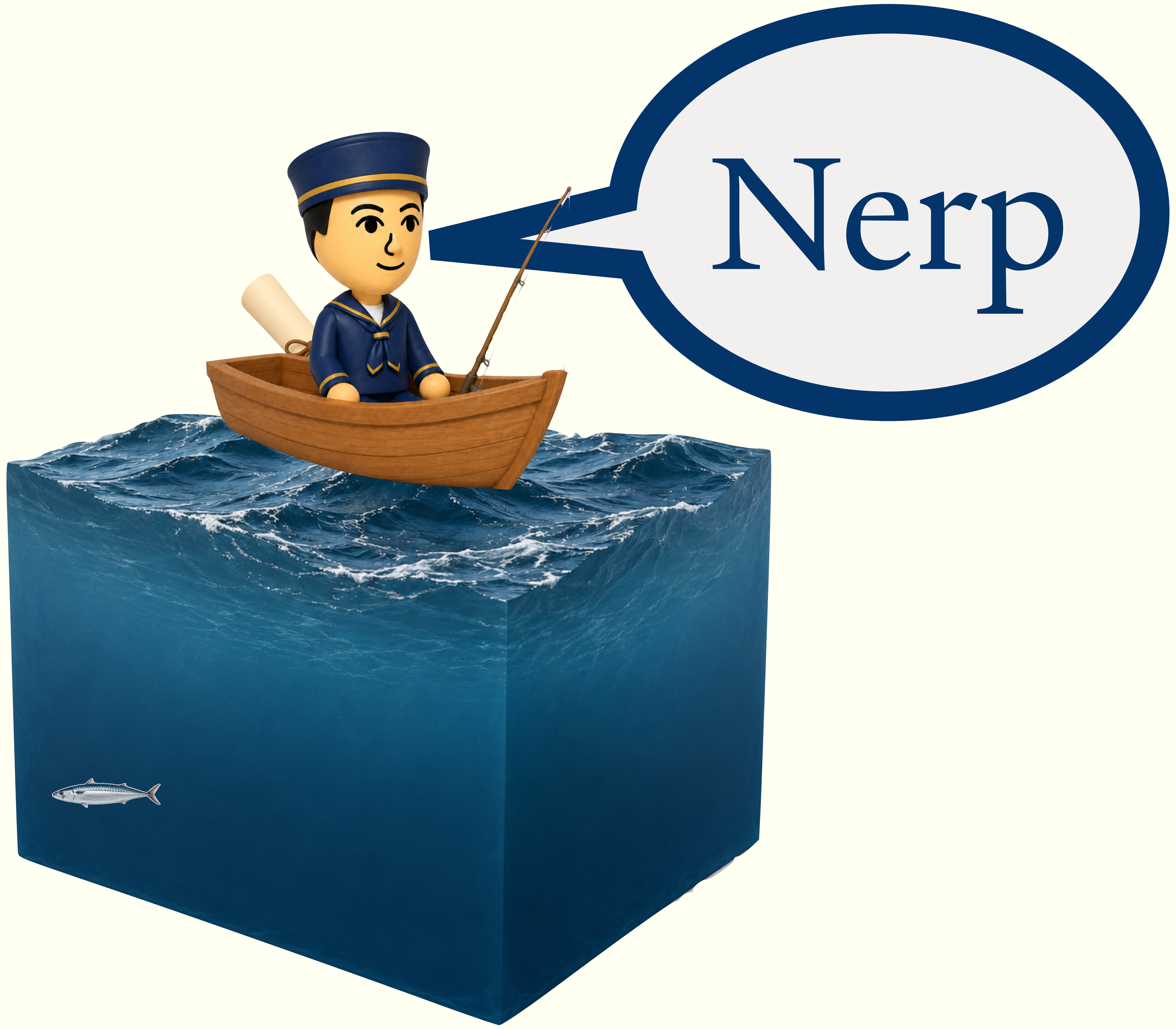
Imagine ``simulating" the problem in a pool: Let the sailor move around as if they were in the ocean, but any time they fish they won't catch anything.

Because they (by assumption) distinguished between the two cases, if we look at where they want to fish, we will likely catch a fish! Say they fished at position x_1…

A new approach for ruling out QCMA

Now, by assumption the should distinguishes between many fish and one fish in the spot he just told us to fish in…





If the sailor doesn't find another fish in a different spot, they can't distinguish them!

A new approach for ruling out QCMA

Now simulate this problem in a pool: Let the sailor move around as if they were in the ocean, but any time they fish they won't catch anything, unless they fish at x_1.

If we look at where they want to fish, we will likely catch a fish in a different location! Say they fished at position x_2…

A new approach for ruling out QCMA

We can keep repeating this process: Simulate the sailor in a pool to get another position where a fish will be, then re-do the simulation with more fake fish!

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We can keep repeating this process: Simulate the sailor in a pool to get another position where a fish will be, then re-do the simulation with more fake fish!

Even if the map contains enough information to tell the sailor about v fish…
We can repeat this simulation to learn where $\gg v$ fish are, without even fishing at all!

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Contradiction!

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If (instead) we had a potion that the sailor had to drink, once they drink it on the first round we wouldn't be able to run the second simulation!

Getting back to QMA

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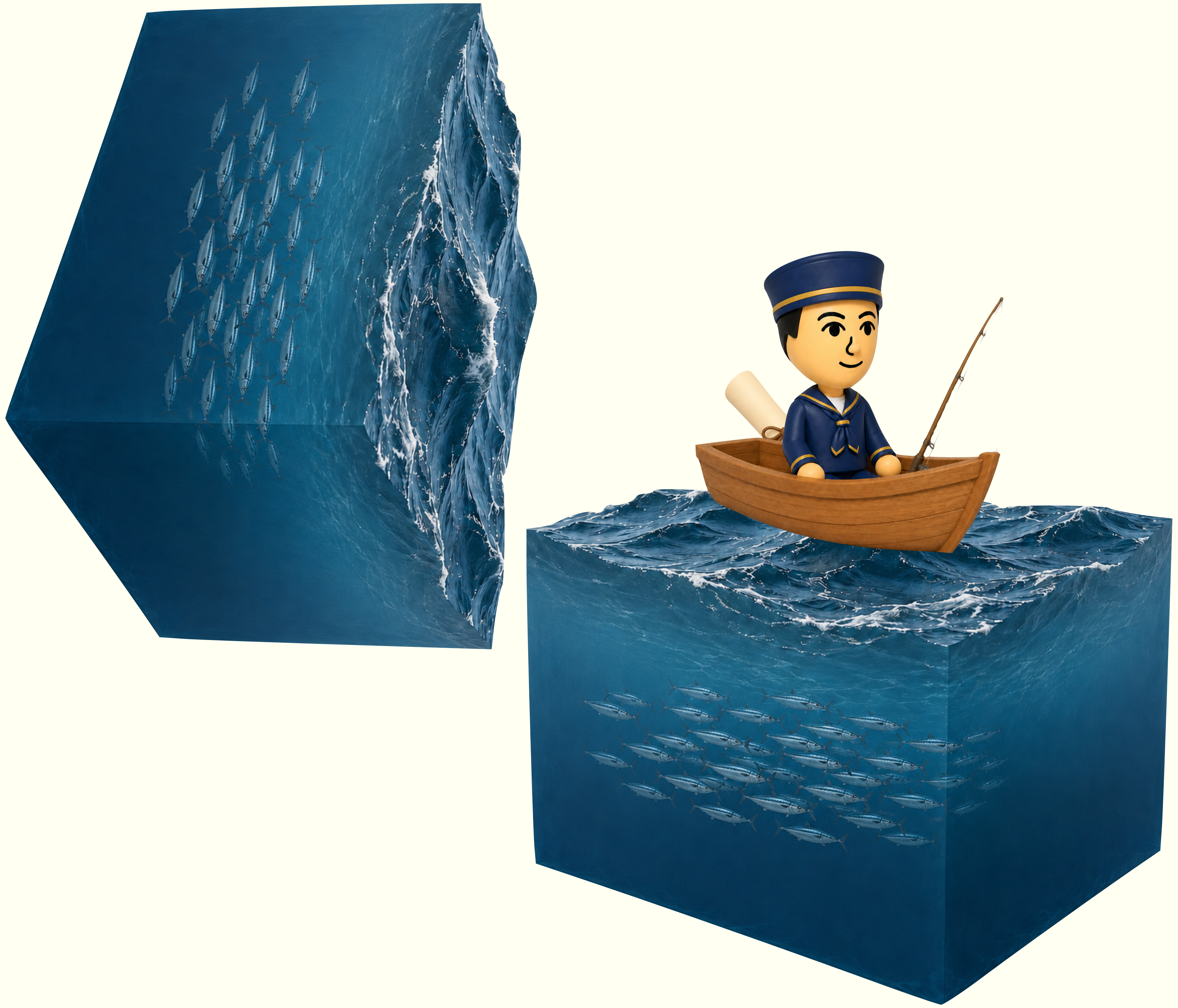
Let's look at two ways of putting the problem back in QMA.

vs

Idea 1: A second oracle [BHNZ’26]

We can add a second oracle that encodes information about the “Fourier transform” of the set S. This results in a problem we call spectral Forrelation.



vs

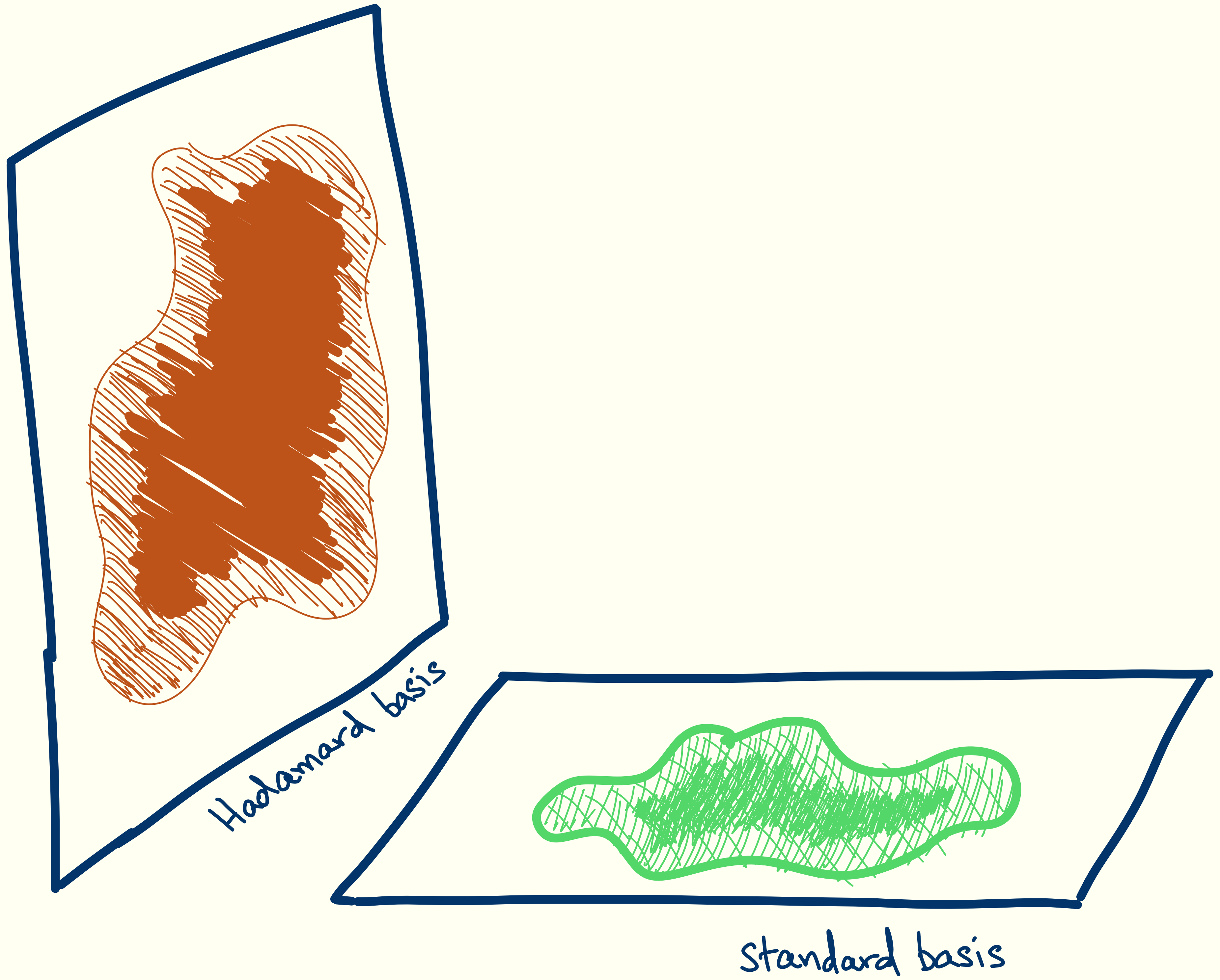
Idea 2: Code intersection [BHV'26]

Another approach will be to add structure to the sparse set S so that a quantum proof can encode many points in the set, but a classical proof can't. This results in a problem called code intersection.

vs

The spectral Forrelation problem

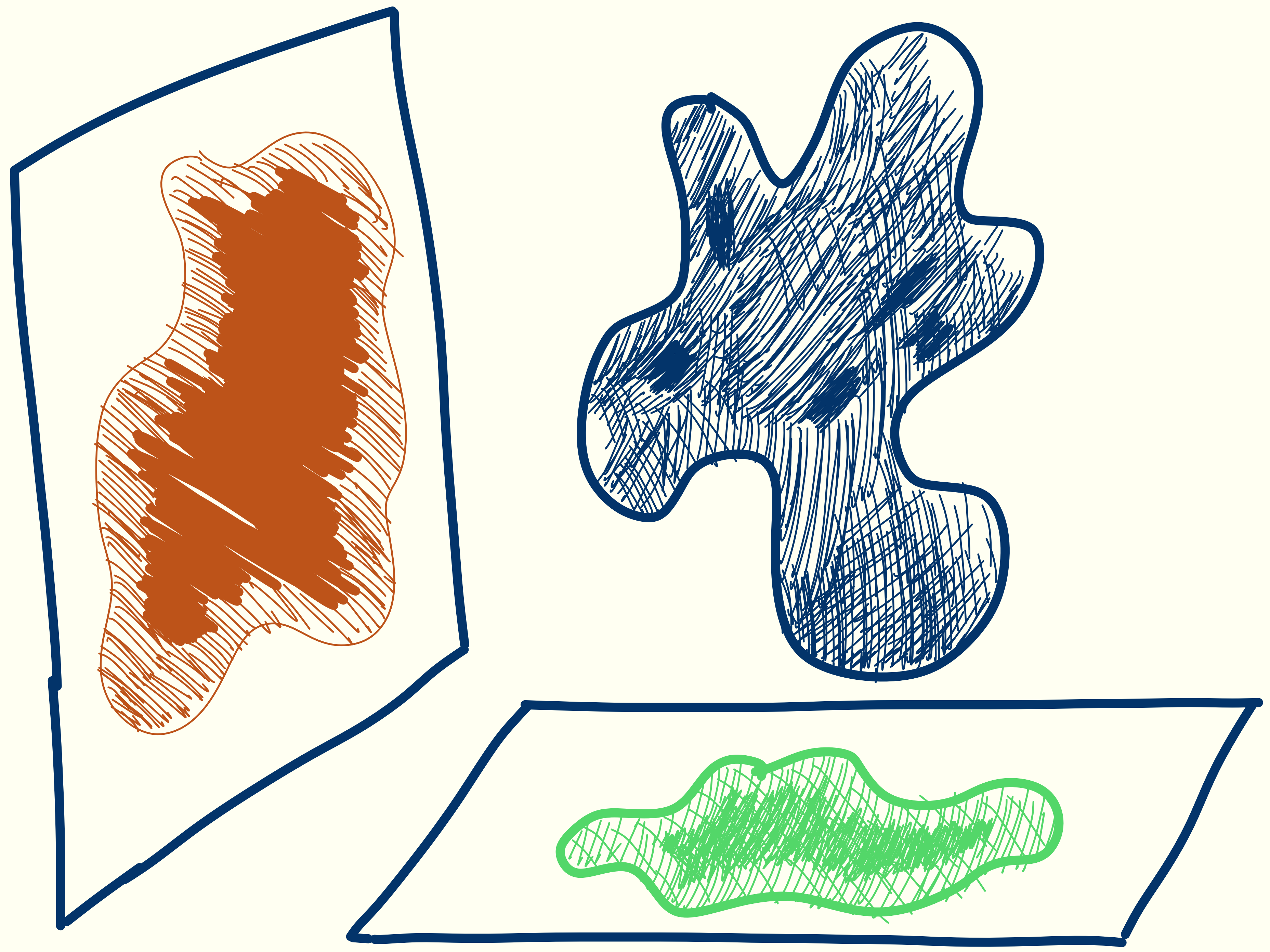
The spectral Forrelation problem is a problem about pairs of sets (S, U), which we treat as oracles through the set membership functions.



The spectral Forrelation problem

We say that two sets (S, U) are spectrally Forrelated if there is a state $|\psi\rangle$ such that

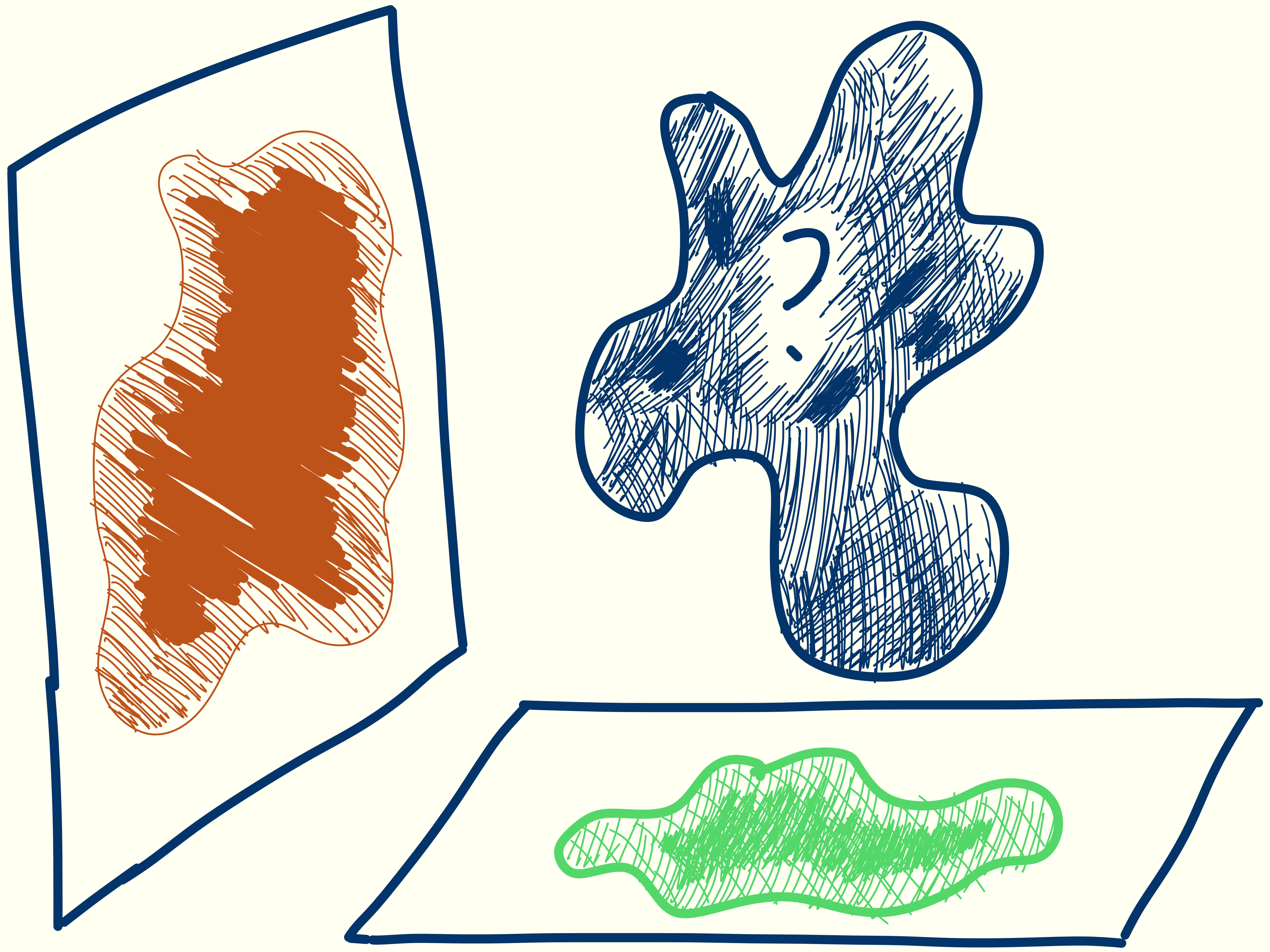
1. $|\psi\rangle$ is mostly supported on S
2. The Fourier transform of $|\psi\rangle$ is mostly supported on U.



The spectral Forrelation problem

Input: Oracle access to two sets (S, U) (via set membership functions)

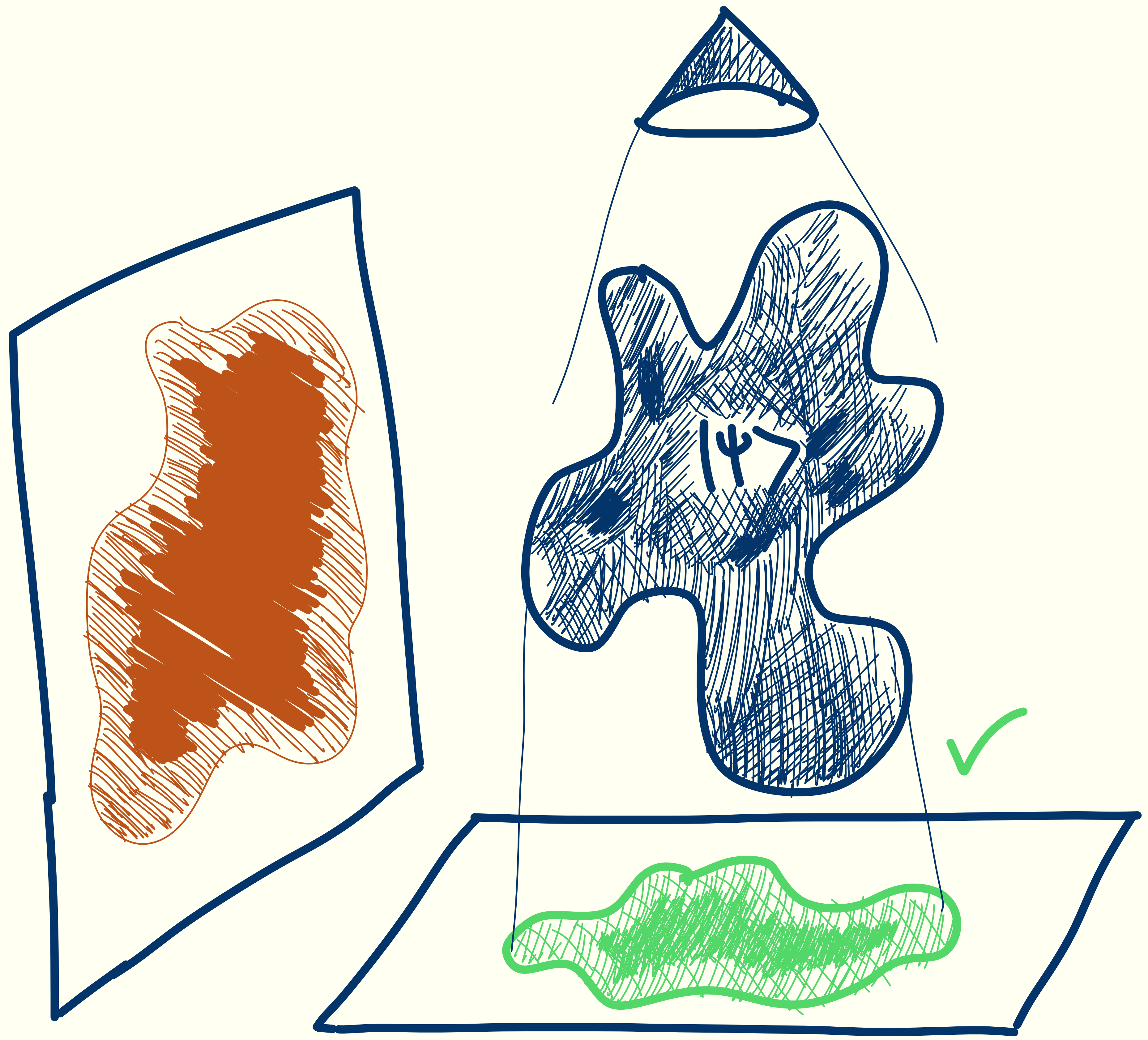
Output: Are they spectrally Forrelated or not?



Spectral Forrelation is in QMA

Given a copy of a state $|\psi\rangle$:

- Check if the state is supported on S, reject if not.



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- Check if the state is supported on S, reject if not.
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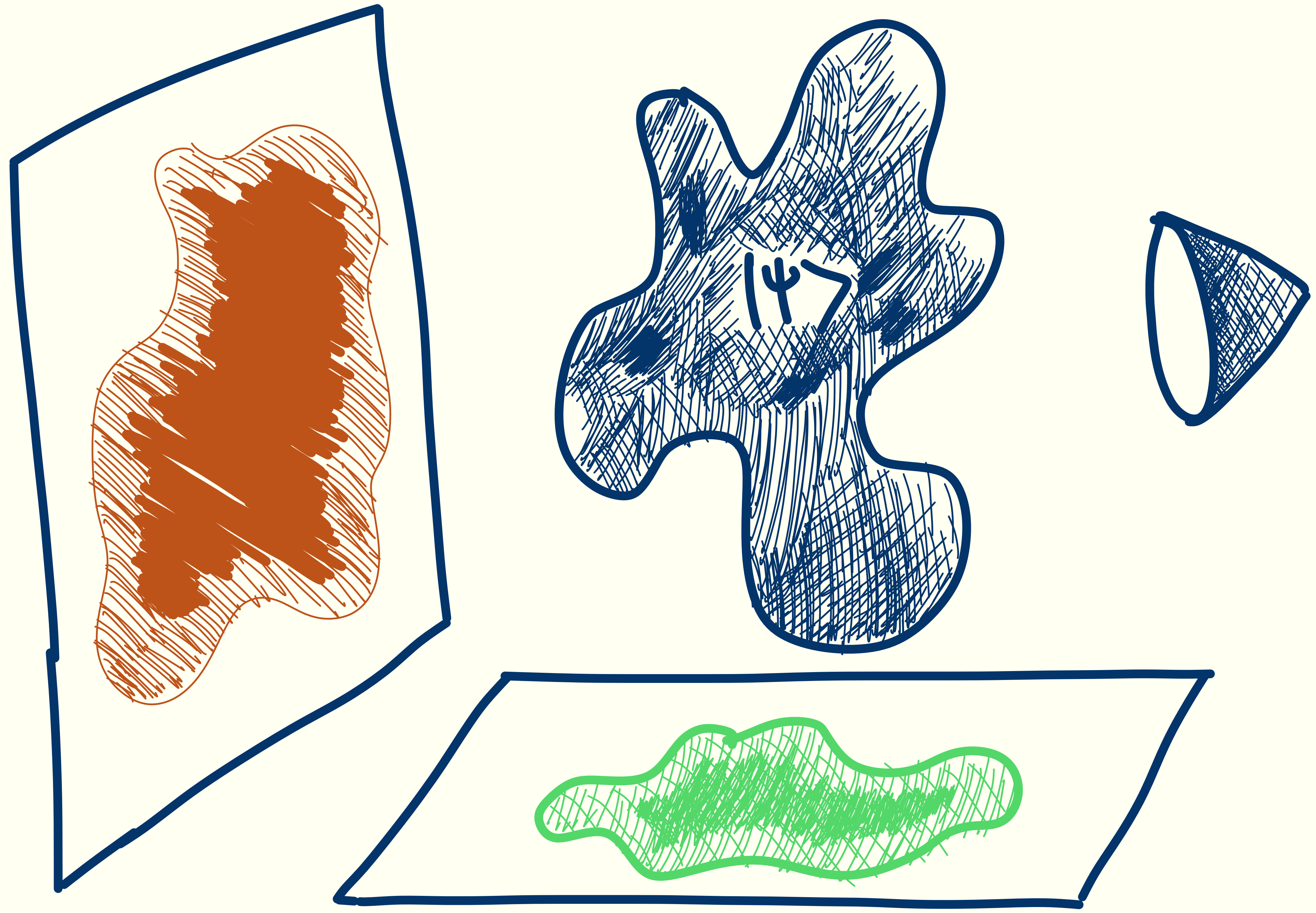
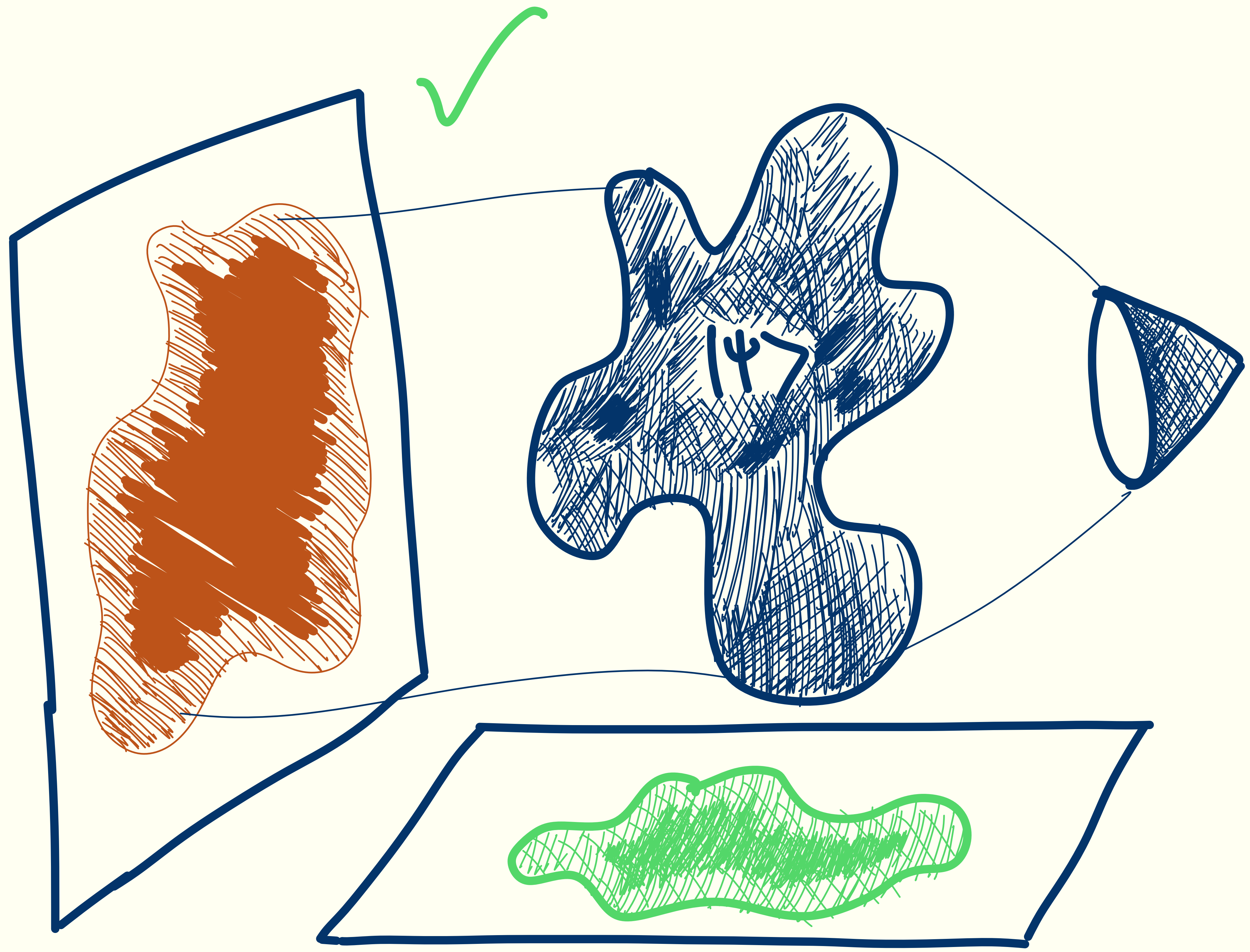


Spectral Forrelation is in QMA

Given a copy of a state $|\psi\rangle$:

- Check if the state is supported on S, reject if not.
- Fourier transform the state.
- Check if the state is supported on U, reject if not.
- Accept.



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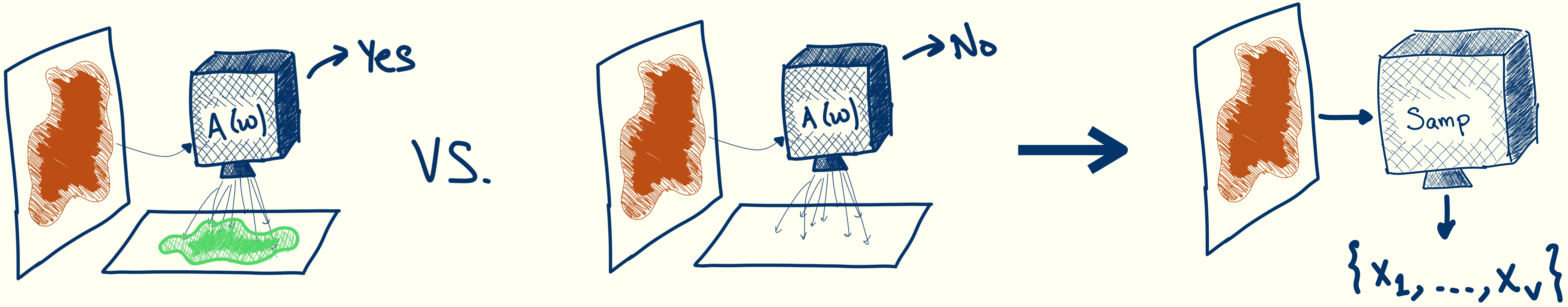
Theorem 1: A QCMA algorithm implies a good sampler that only looks at U.



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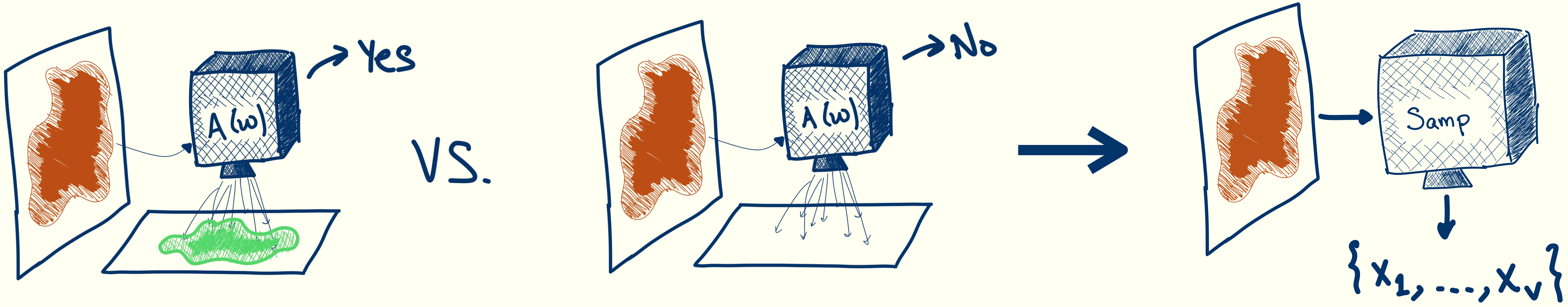

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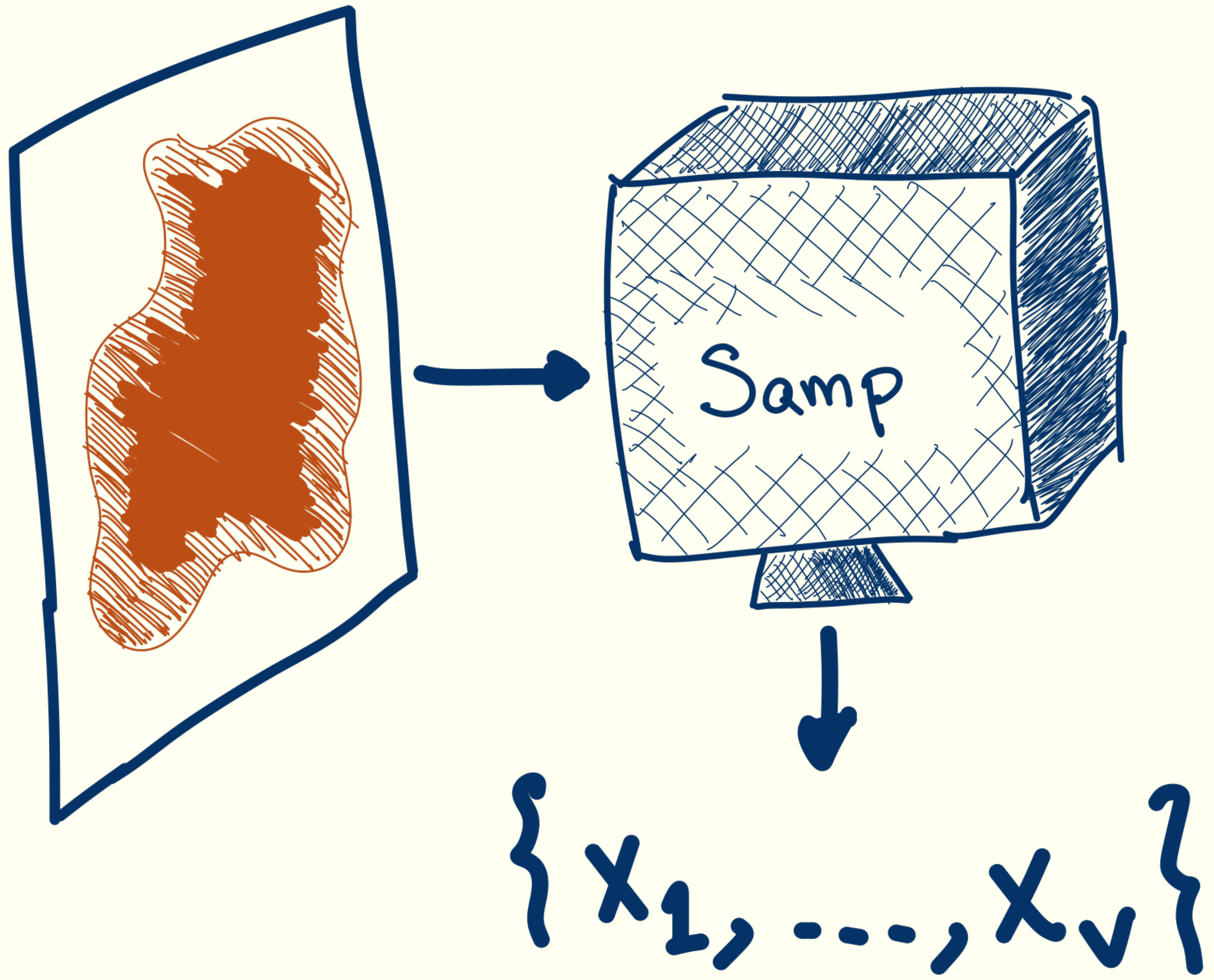

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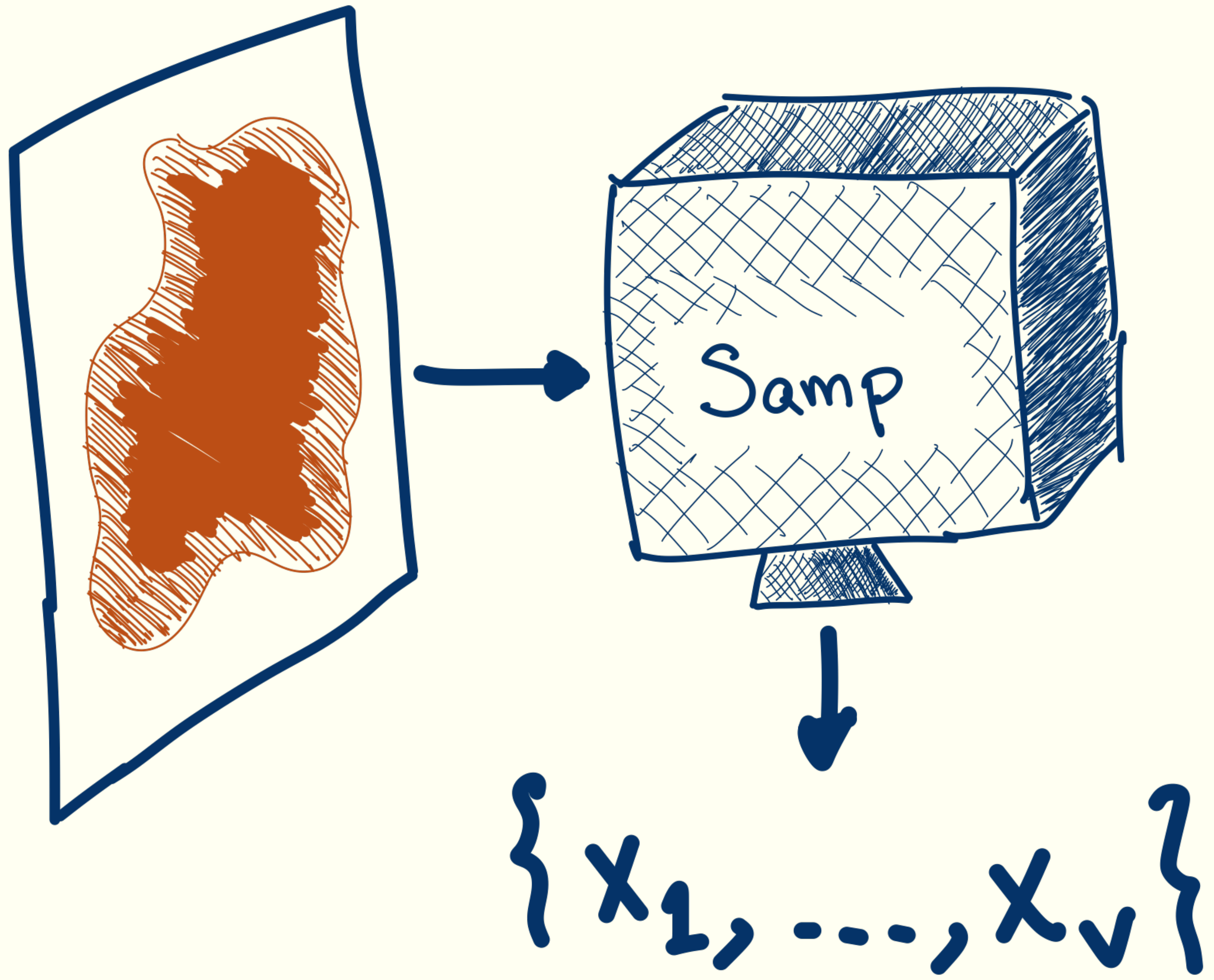

This is more tricky than before: U can be correlated to S.
→ We need to understand what the sampler knows about S when they look at U!

Lazy sampling

Classically, when someone interacts with a random function, we don't write down every entry. Lazy sampling is how we only store what the adversary knows about the function.

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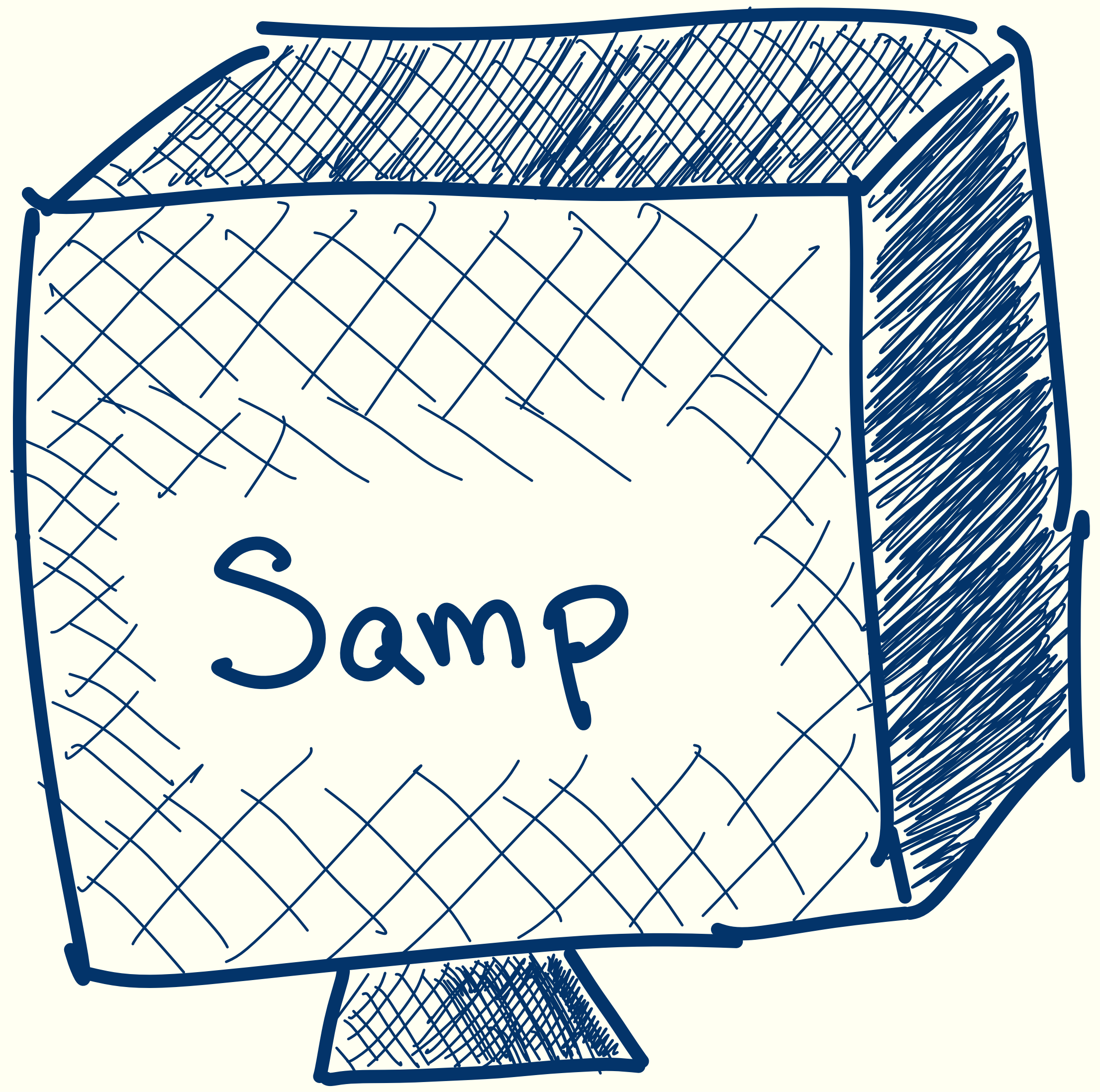
z	1
w	0



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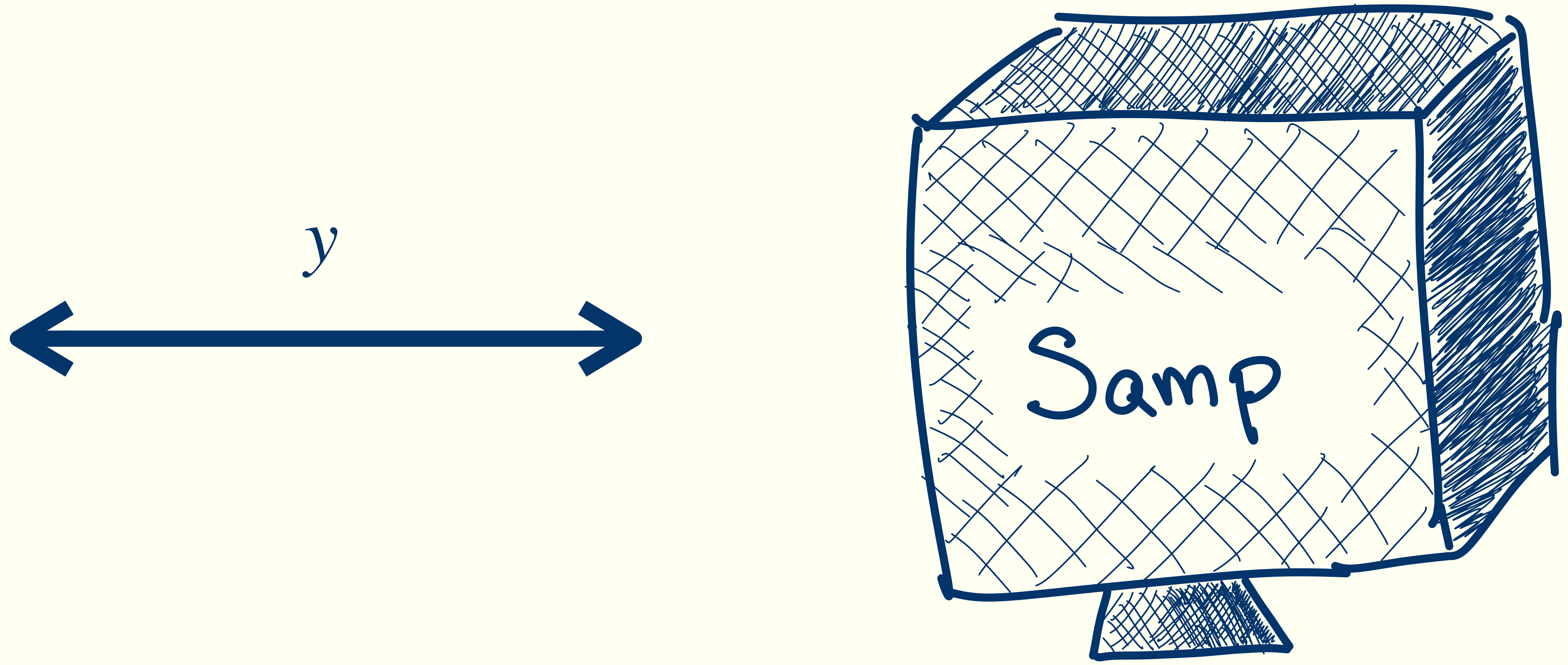

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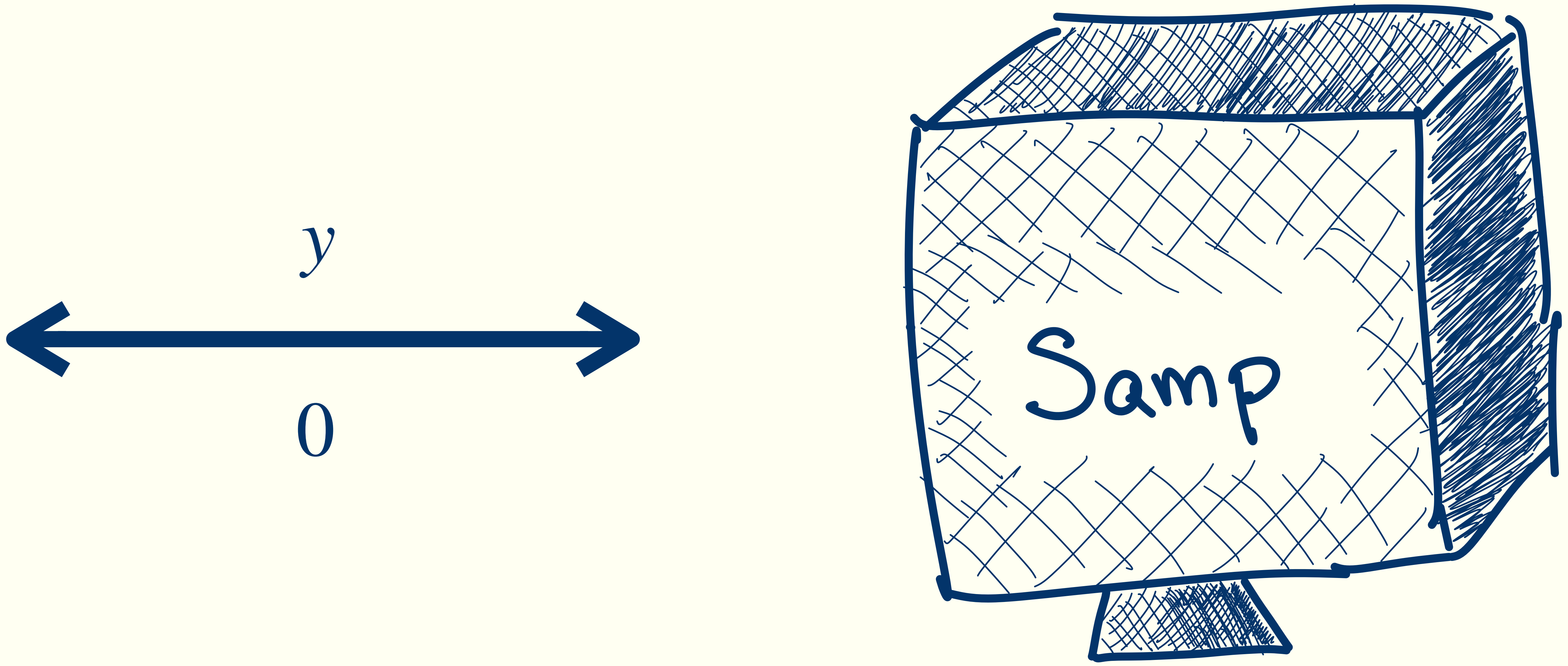

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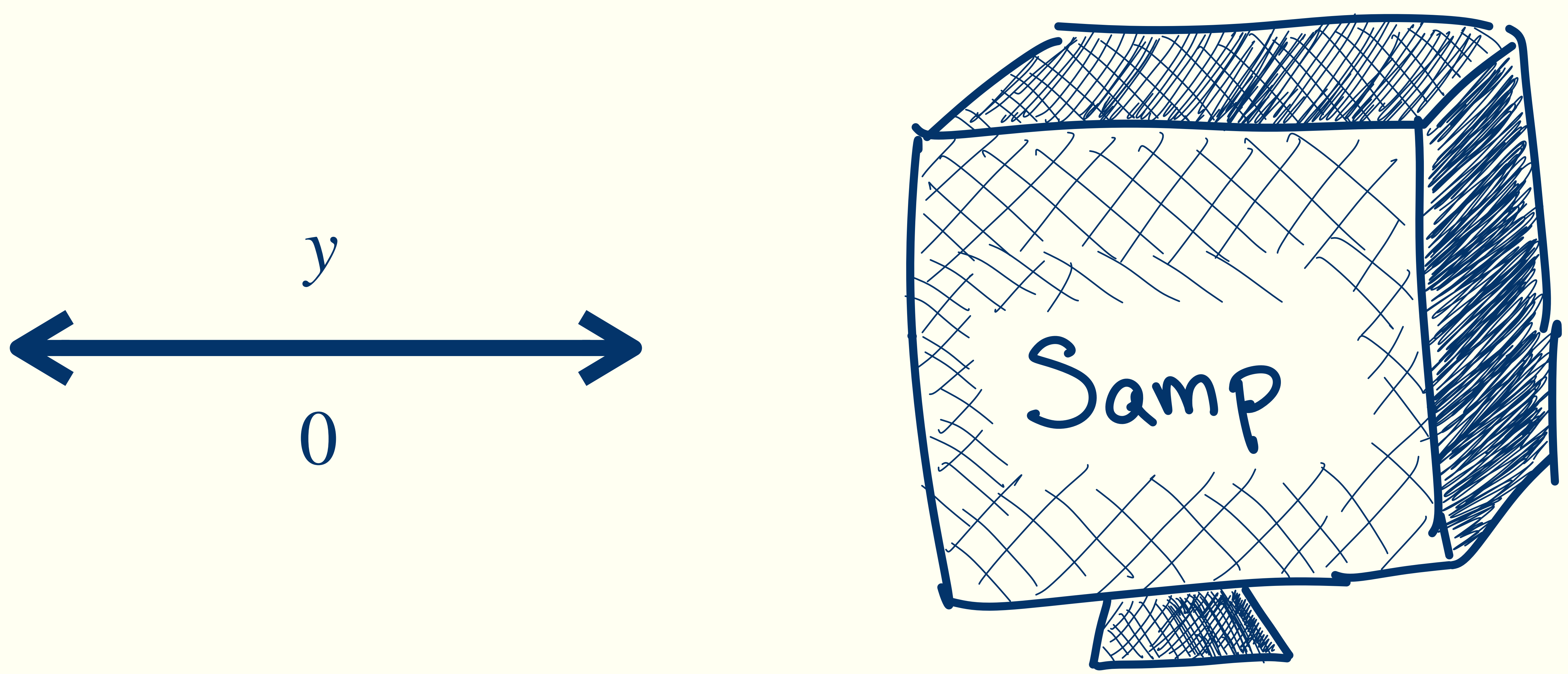
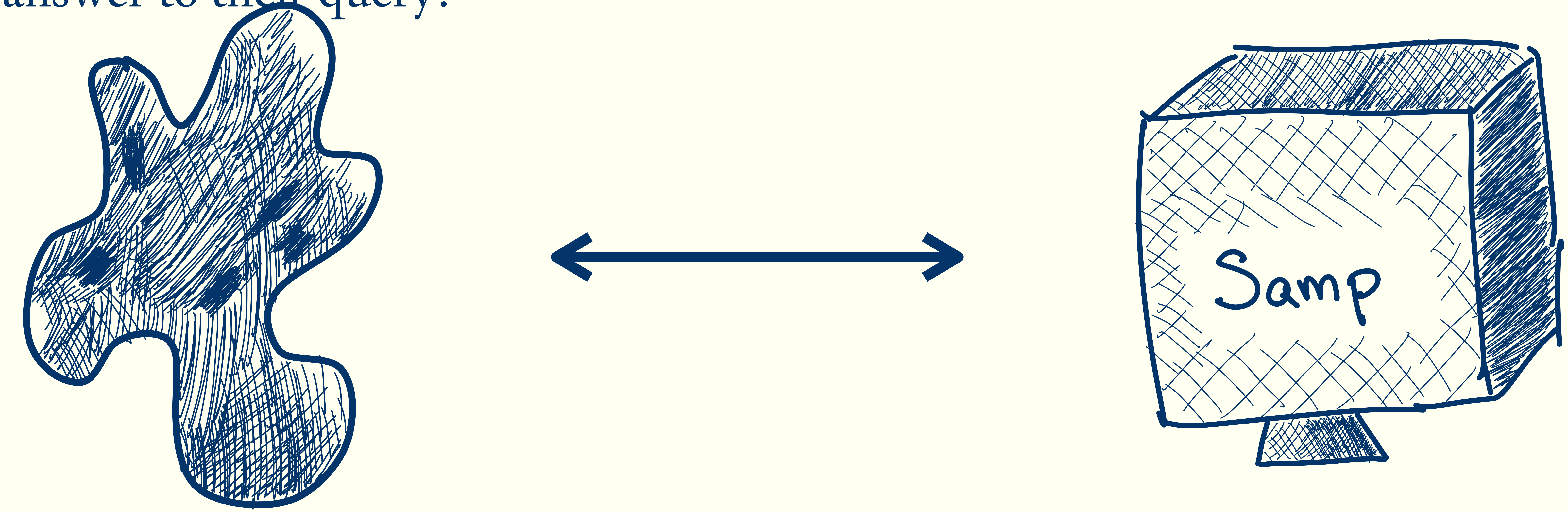


A similar technique exists quantumly, called the compressed oracle technique.

Compressed oracles

In the compressed oracle technique, the sampler interacts with an unknown quantum system. When they query, we instead "condition" on the system being consistent with the answer to their query!



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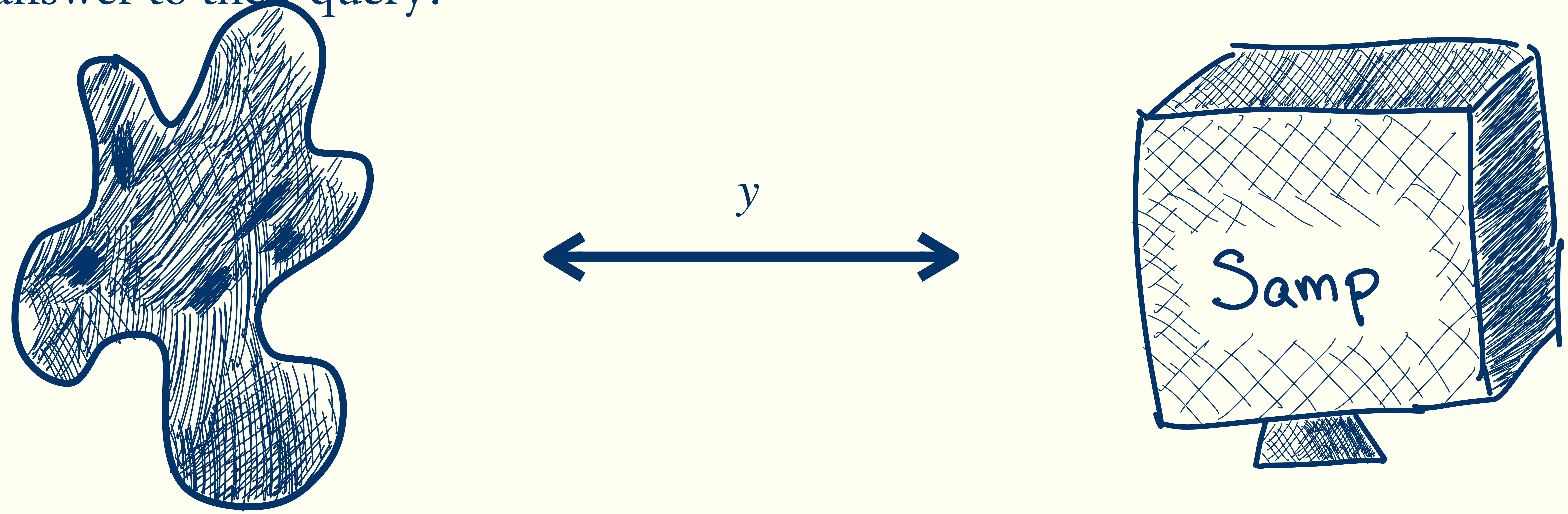

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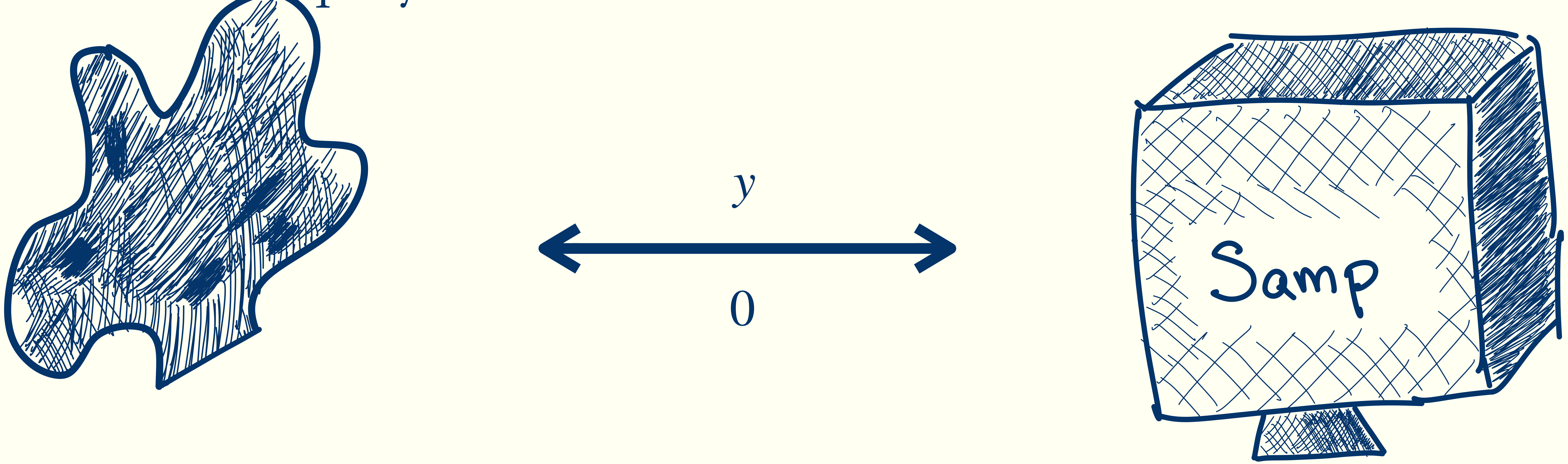
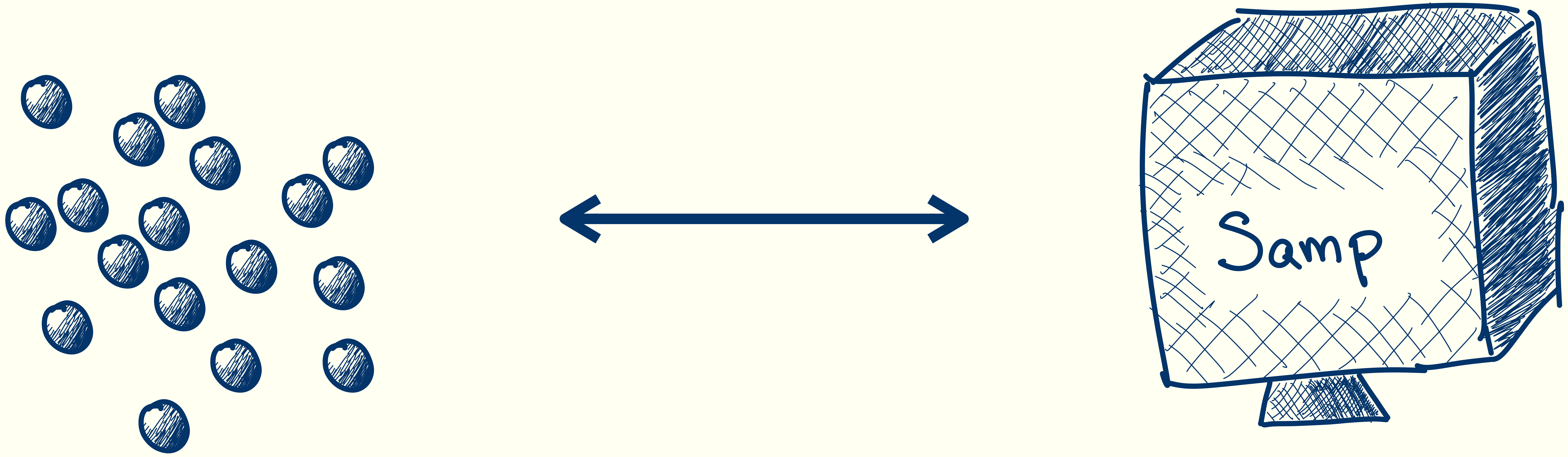


Compressed oracles for spectral Forrelation

It turns out that doing this for spectral Forrelation leads to an interesting connection with physics! We model the quantum system is as a collection of ``bosons” (like, photons or gluons from physics) at random positions…



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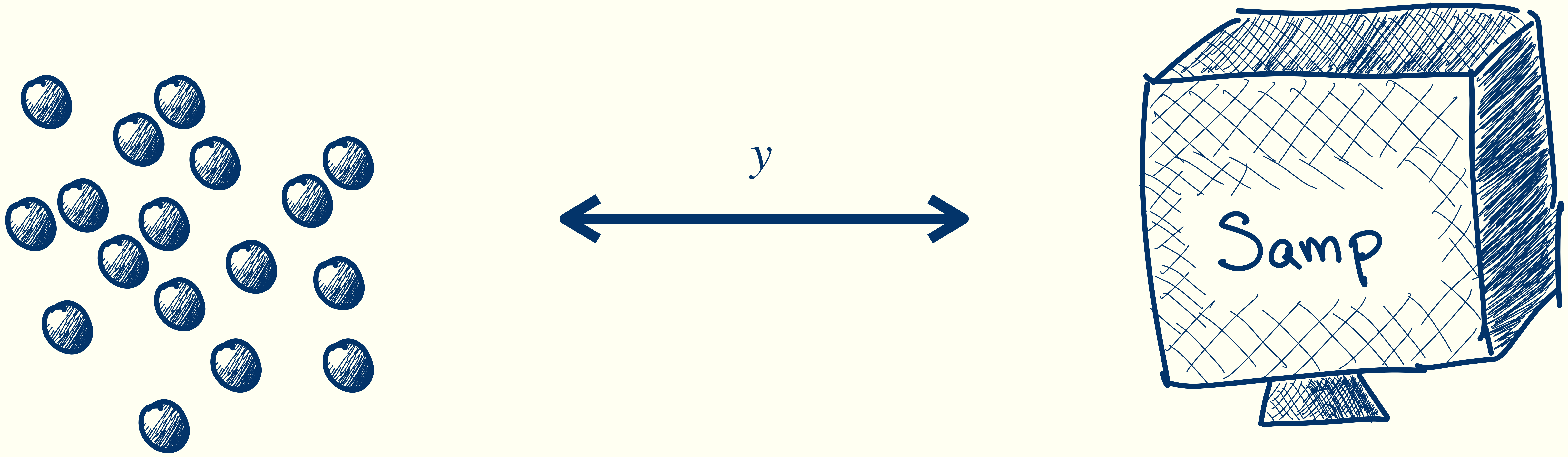

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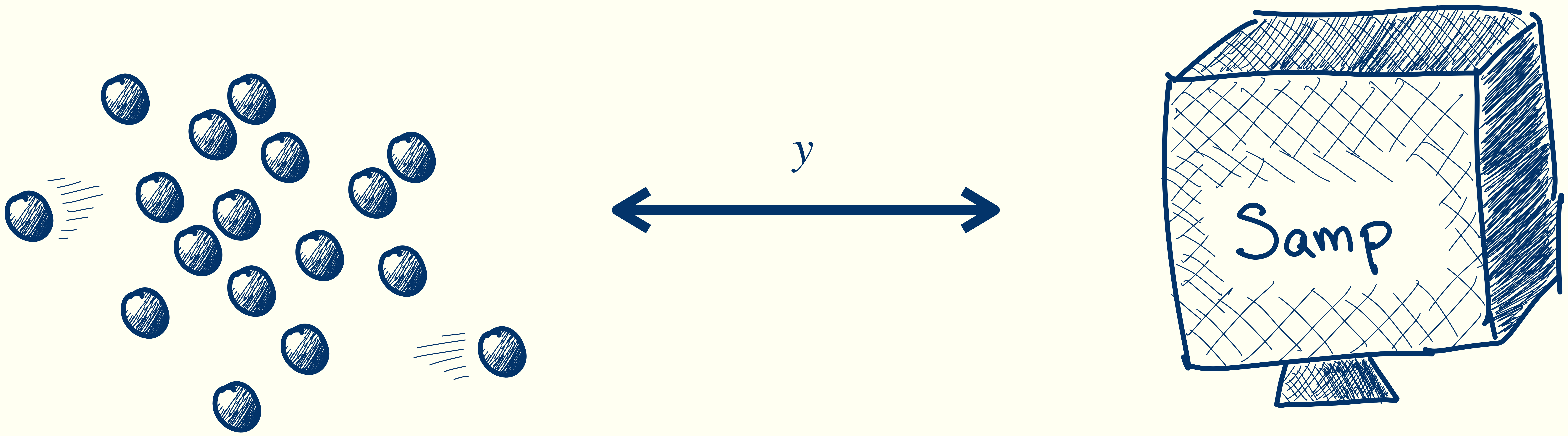

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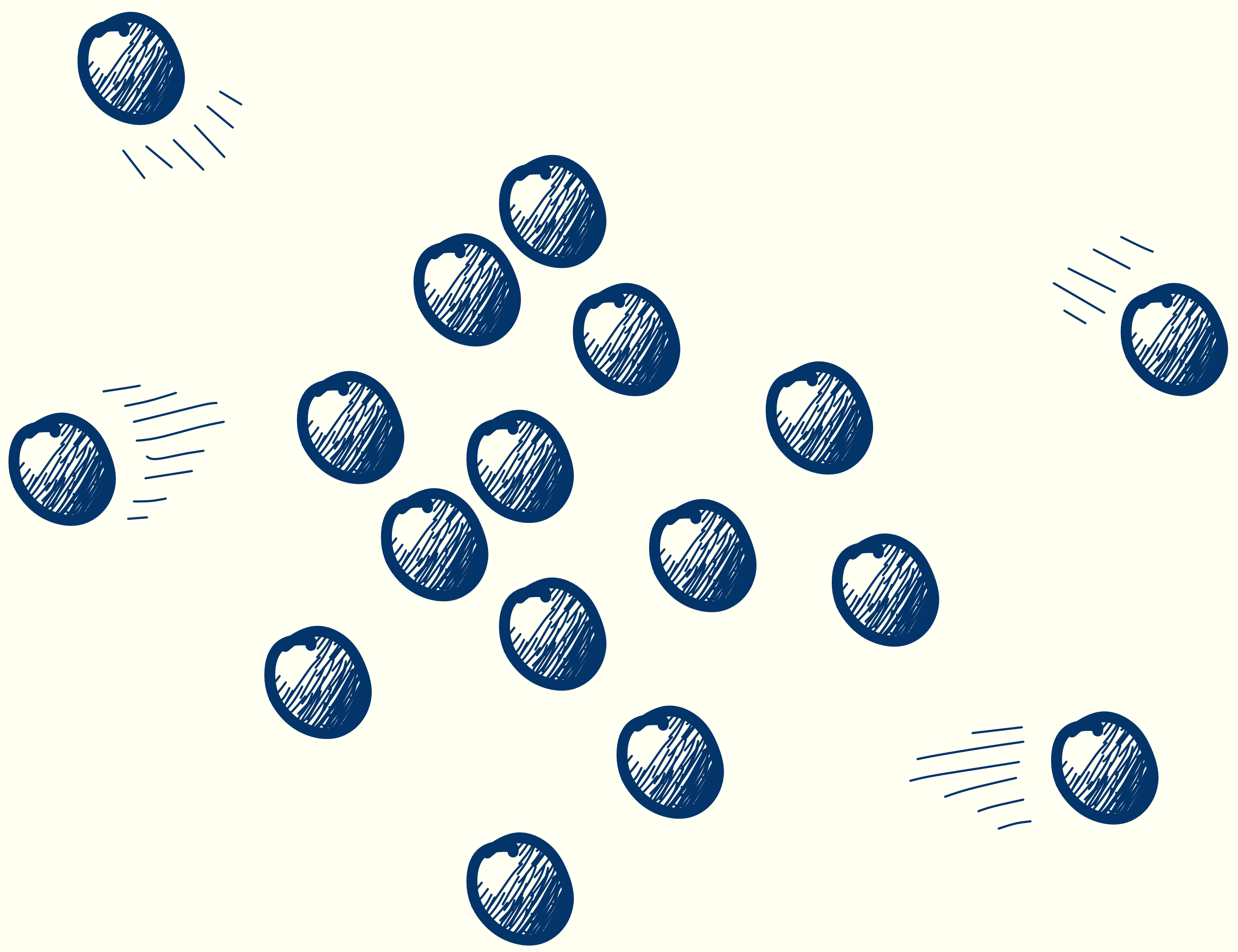
Why can't QCMA solve spectral Forrelation?

We show that knowing about the momentum of pairs of bosons tells you nothing about the positions of those bosons. As long as the bosons remain paired up, you can't find their positions.



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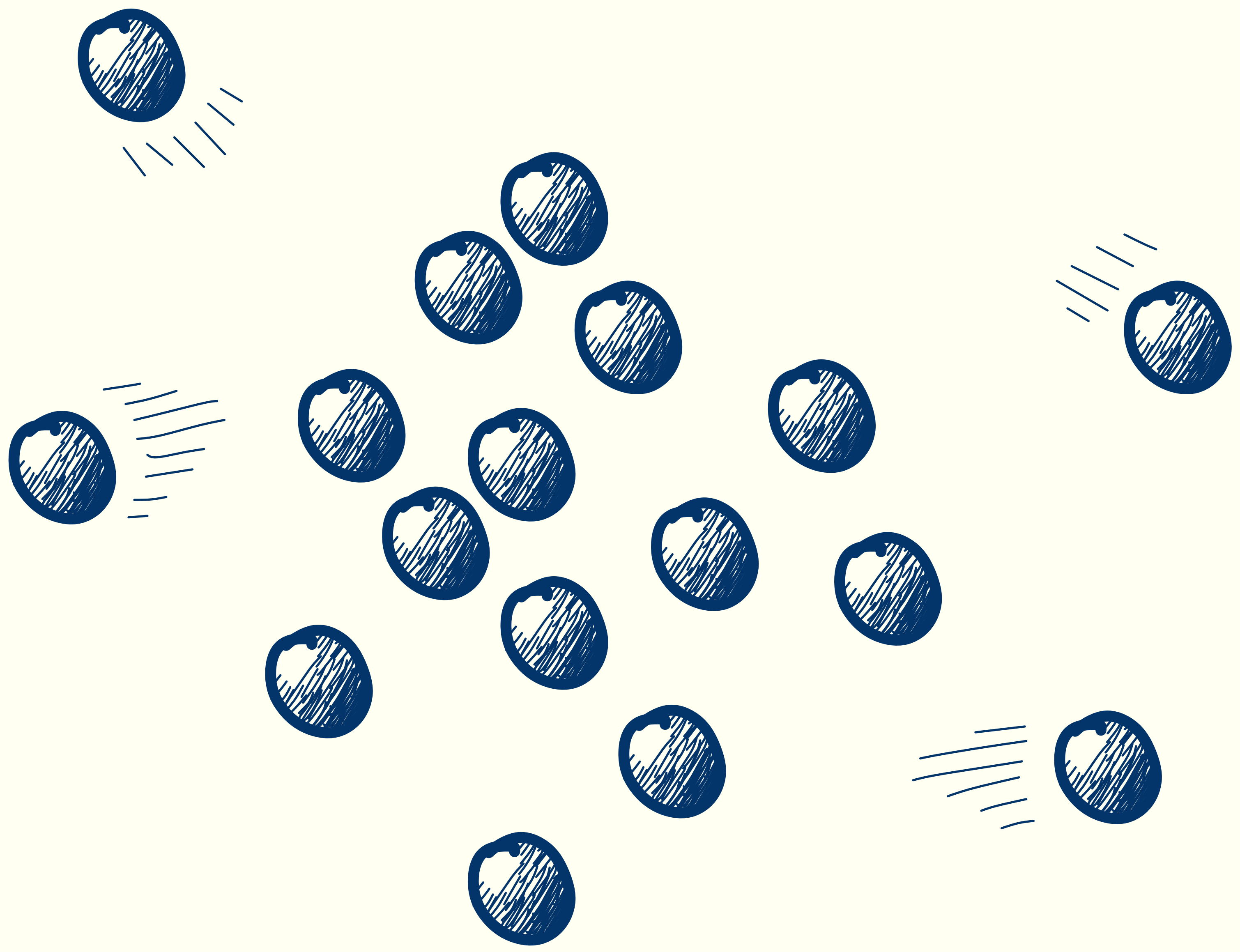

Not everything is always paired: the sampler might ``double bounce'', i.e., add momentum to a boson that already was moving. But using fancy math, we can show this almost never happens → The sampler implies a contradiction!

Takeaways and future directions

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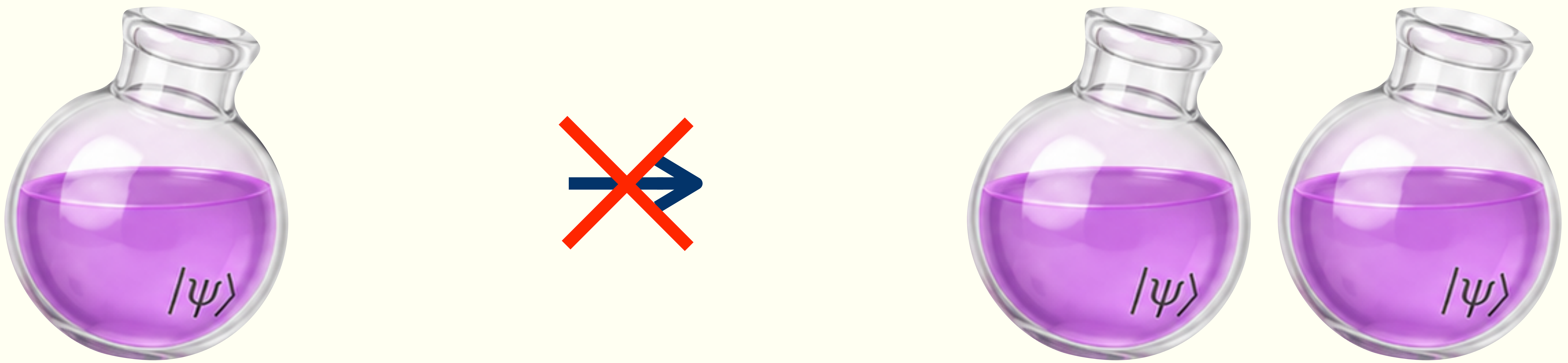

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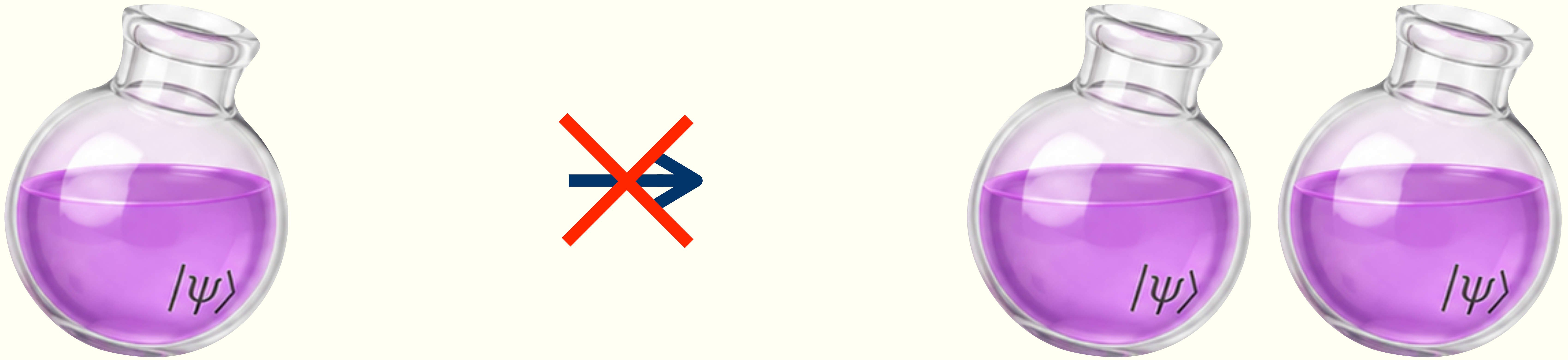

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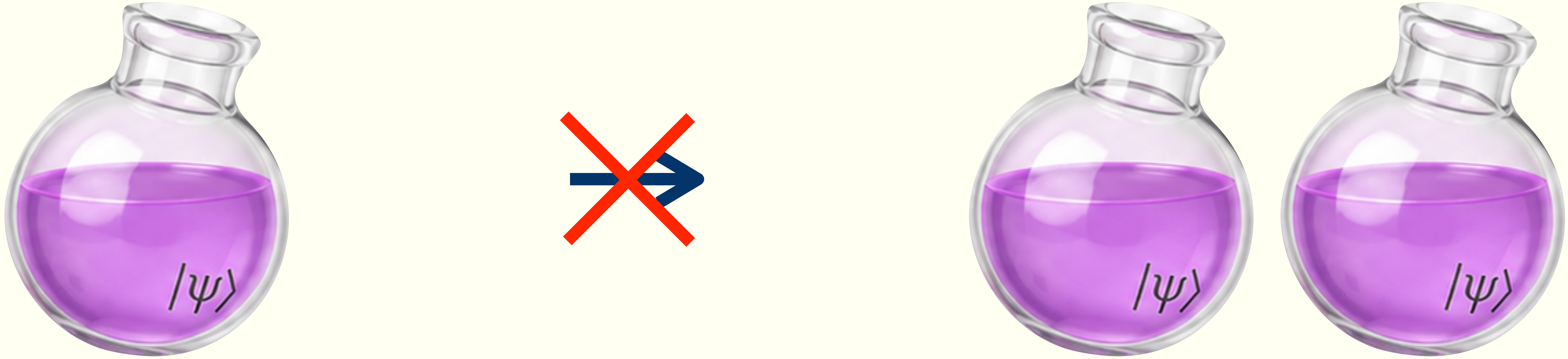

Unfortunately, this power actually prevents us from making them clonable! Our ideas show that there are some problems that can only be proven once, otherwise they would lead to impossible situations!

Takeaways and future directions

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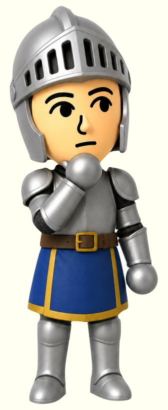

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These problems are still really hard though…

Thank you to everyone who took part in my journey!